Physics 214 Fall 2002
Assignment #10 (due 11/21)

Reading: *Young & Freedman* sections 37–2,3,4,6; 38–1,2,3,4,5,6.

1. *Young & Freedman* Problem 38-61. Add:

   (f) Show that there are $N-2$ secondary ("lesser") maxima between each pair of principal maxima.

   (g) Plot the intensity as a function of ϕ for $N=4$.

   (h) Explain the advantages of making N large in a grating.

2. A laser beam (wavelength λ) is shone on a *wide slit* (width $a=\lambda$), so that the angle between the beam and the normal to the slit plate is $\alpha = 20°$ (see Figure 1).

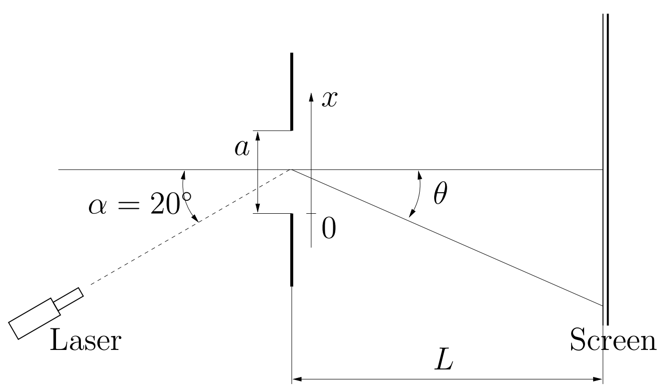

Figure 1: Wide slit with an off-axis source.

(The laser is far away from the slit, so you can assume that the light arrives there in the form of *plane waves.*)

   (a) Because the light waves from the laser have to travel different distances to each point of the slit, the points along the slit act like *point sources with different phases.* Find the phase $\Phi(x)$ of a spherical wavelet emerging from a point at a distance x from the bottom of the slit, relative to the point at $x=0$.

   (b) Derive a formula for the intensity $I(\theta)$ of the diffraction pattern formed on a screen placed at a distance $L \gg a$ behind the slit. *Hint:* As done in class, split the finite slit into N infinitesimal slits distance d apart (i.e., at positions $x = 0, d, 2d, \ldots$) and derive an N-slit interference formula. Then take the limit $N \to \infty$, $d \to 0$ with $Nd = a$.

   (c) Identify the angles (θ) of the principal maxima and all the minima and sketch a plot of the diffraction pattern on the screen.

3. *Alternate derivation of the single-slit pattern.* Instead of assuming a finite number N of sources and taking the limit $N \to \infty$, we just integrate. Assume the slit has width a, extending from $x = -a/2$ to $x = +a/2$ instead of from $x = 0$ to $x = a$ as in Figure 1. (You get the correct answer either way, but this way makes the math slightly easier.) The pattern is viewed on a distant screen. The slit is illuminated with plane waves normal to the slit (i.e. $\alpha = 0$ in Figure 1). The electric field at the screen due to an infinitesimal slice of the slit from x to $x + \Delta x$ is

$$dE = \frac{E_0}{a} dx\ e^{ik\Delta r} \tag{1}$$

Here, E_0 is the electric field amplitude we would use for the entire slit, E_0/a is the electric field per unit width, and $(E_0/a)dx$ is the (infinitesimal) electric field amplitude at the screen due to a slice of width dx.

(a) Explain (in words) what Δr represents. Assuming that $\Delta r = 0$ for $x = 0$, what is Δr in terms of x and θ?

(b) Integrate from $x = -a/2$ to $x = +a/2$ to add up the contributions to the amplitude at the screen from every slice . What is the amplitude at the screen as a function of θ? What is the intensity at the screen as a function of θ?

4. *Young & Freedman* Problem 38-53.