Reading: Young & Freedman sections 37–2,3,4,6; 38–1,2,3,4,5,6.

1. Young & Freedman Problem 38-61. Add:
   
   (f) Show that there are \( N - 2 \) secondary (“lesser”) maxima between each pair of principal maxima.
   
   (g) Plot the intensity as a function of \( \phi \) for \( N = 4 \).
   
   (h) Explain the advantages of making \( N \) large in a grating.

2. A laser beam (wavelength \( \lambda \)) is shone on a wide slit (width \( a = \lambda \)), so that the angle between the beam and the normal to the slit plate is \( \alpha = 20^\circ \) (see Figure 1).

   (a) Because the light waves from the laser have to travel different distances to each point of the slit, the points along the slit act like point sources with different phases. Find the phase \( \Phi(x) \) of a spherical wavelet emerging from a point at a distance \( x \) from the bottom of the slit, relative to the point at \( x = 0 \).

   (b) Derive a formula for the intensity \( I(\theta) \) of the diffraction pattern formed on a screen placed at a distance \( L \gg a \) behind the slit. \textit{Hint: } As done in class, split the finite slit into \( N \) infinitesimal slits distance \( d \) apart (i.e., at positions \( x = 0, d, 2d, \ldots \)) and derive an \( N \)-slit interference formula. Then take the limit \( N \to \infty, \ d \to 0 \) with \( Nd = a \).

   (c) Identify the angles \( (\theta) \) of the principal maxima and all the minima and sketch a plot of the diffraction pattern on the screen.
3. Alternate derivation of the single-slit pattern. Instead of assuming a finite number $N$ of sources and taking the limit $N \to \infty$, we just integrate. Assume the slit has width $a$, extending from $x = -a/2$ to $x = +a/2$ instead of from $x = 0$ to $x = a$ as in Figure 1. (You get the correct answer either way, but this way makes the math slightly easier.) The pattern is viewed on a distant screen. The slit is illuminated with plane waves normal to the slit (i.e. $\alpha = 0$ in Figure 1). The electric field at the screen due to an infinitesimal slice of the slit from $x$ to $x + \Delta x$ is

$$dE = \frac{E_0}{a} dx \ e^{ik\Delta r} \tag{1}$$

Here, $E_0$ is the electric field amplitude we would use for the entire slit, $E_0/a$ is the electric field per unit width, and $(E_0/a)dx$ is the (infinitesimal) electric field amplitude at the screen due to a slice of width $dx$.

(a) Explain (in words) what $\Delta r$ represents. Assuming that $\Delta r = 0$ for $x = 0$, what is $\Delta r$ in terms of $x$ and $\theta$?

(b) Integrate from $x = -a/2$ to $x = +a/2$ to add up the contributions to the amplitude at the screen from every slice. What is the amplitude at the screen as a function of $\theta$? What is the intensity at the screen as a function of $\theta$?