1 Problem 1: Electromagnetic Wave [14 points]

An electromagnetic wave hits a detector at t = 0. It reads $\vec{E} = E_0 \hat{x} - E_0 \hat{y}$ (E > 0) and $\vec{B} = B\hat{z}$ (B > 0).

(a) (4 points)

Draw the vectors \vec{E} and \vec{B} .



(b) (4 points)

What is B in terms of E_0 ?

(c) (4 points)

What is the direction the wave is traveling? Draw a vector indicating this direction on the coordinate system provided and specify whether it is in the xy, xz, or yz plane.



(d) (2 points)

Is the wave *linearly (plane)* or *circularly* polarized?

- (A) Linearly.
- (B) Circularly.
- (C) Not enough information.

2 Problem 2: Polarization of Microwave [12 points]

Recall the microwave lab you did. Suppose a microwave traveling to the right (+x-direction) gives a reading on the receiver of intensity I_0 . Now, a polarization grid oriented in the vertical (y-) position is inserted between the generator and the receiver. (See Figure 1.)



(a) (4 points)

The receiver reading is zero, that is, the signal is completely screened by the polarization grid in the vertical position. In terms of the *xyz*-coordinate system given in the figure, *what is the direction of the electric field of the microwave as it exits the generator?*

(b) (4 points)

Suppose you insert a second polarization grid at a 45° angle in the *y*-*z* plane between the *microwave* generator and the vertical polarization grid. What should the reading on the receiver be?

- (A) I_0
- (B) zero
- (C) $I_0/2$
- (D) $I_0/4$
- (E) $I_0/\sqrt{2}$

(c) (4 points)

If, instead, the second polarization grid at a 45° angle is placed between the the vertical polarization grid and the receiver, what is the reading now?

(A) I_0

(B) zero

- (C) $I_0/2$
- (D) $I_0/4$
- (E) $I_0/\sqrt{2}$



Figure 2: Closed tubes filled with noble gases.

3 Problem 3: Standing Waves in Sound Tubes [8 points]

Four tubes, each with two closed ends, are filled with different noble gases at atmospheric pressure and room temperature; therefore, they have the same bulk modulus B but different mass densities ρ_0 , $2\rho_0$, etc. (see Figure 2). The tubes also have different lengths, as shown on Figure 2. Which two of these (A & B or B & D, etc.) tubes will have the same lowest resonant frequency? Provide your answer in the box below.



Figure 3: Generalized boundary conditions for a sound tube.

4 Problem 7: Generalized Boundary Conditions for a Sound Tube [14 points]

A tube of length L is filled with air of bulk modulus B and mass density ρ_0 at atmospheric pressure. One end of the tube (x = L) is closed while the other end (x = 0) is attached to a massless piston of area A that can slide freely (without friction) along the tube. (We consider low amplitude waves so that you can take $L \approx \text{const.}$) The piston is attached to an ideal spring of spring constant k that exerts a horizontal force on the piston; the spring is relaxed when the piston is at x = 0. (See Figure 3.) We denote the sound displacement inside the tube as S(x, t) and the pressure inside the tube as P(x, t).

(a) Air Pressure and Displacement of the Piston (7 points)

Draw a free body diagram for the piston, indicating the directions and the magnitudes of all the forces, using no quantities other than P(x = 0, t), S(x = 0, t), k, A, ρ_0 , L, and P_0 .

(b) Boundary Condition at the Piston (7 points) Challenge Problem!

Write the Equation of Motion for the piston in terms of the degrees of freedom, i.e., using no quantities other than the displacement function S(x, t) and its derivatives evaluated at $x = 0, k, A, B, \rho_0, L$, and P_0 .