PHYS214
Prelim I
October 6 1998

Name:____

Signature:____

Section # and TA:__________________________________

INSTRUCTIONS

- The exam is out of **100** points.
- By signing this exam you certify to adhere to the Cornell academic integrity code.
- You **CAN NOT** use a calculator nor bring other materials written or otherwise to aid you in this exam. A formulae sheet will be provided.
- Check that you have all **17** pages (including cover page). The formulae sheet is distributed separately.
- You **MUST** place your answer to a Multiple Choice question in the box provided.

MC 1	/5
MC 2	/5
MC 3	/5
MC 4	/5
MC 5	/5
SA 1	/20
SA 2	/25
SA 3	/30
Total	/100

PLEASE TURN PAGE

MC 1 **Wave Equation** **[5 points]**

A traveling wave is described by the equation

$$y(x,t) = 2\cos(2x + 5t)$$

where the displacement y is measured in centimeters, the position x in meters and the time in seconds.

The wave velocity is

(A) 0.4 ms^{-1} to the left

(B) 0.4 ms^{-1} to the right

(C) 2.5 ms^{-1} to the left

(D) 2.5 ms^{-1} to the right

(E) π ms^{-1} to the right

Answer

MC 2 **Power** **[5 points]**

A string is under a tension of 1 N. Its transverse displacement $y(x,t)$, measured in meters, has the following form

$$y(x,t) = 0.4\cos(\pi x)\sin\left(\frac{\pi}{4}t\right)$$

Which of the following is the correct numerical value for the power $P(x,t)$ (measured in Watts) at $x = 0.25$ m and $t = 1$ s?

(A) $\frac{\pi^2}{10}$

(B) $\frac{\pi^2}{100}$

(C) π^2

(D) $\frac{\pi^2}{25}$

(E) $\frac{\pi^2}{50}$

Answer

MC 3 **Numerical Approximation** **[5 points]**

The displacement of an object moving in the z-direction is measured at two times to be

$$z(2.9 \text{ s}) = 0.30 \text{ m}$$

and

$$z(3.0 \text{ s}) = 0.35 \text{ m}\,.$$

The equation of motion of the object is given by

$$\frac{d^2 z}{dt^2} = 6\, z\,.$$

From the information given and the numerical technique you used to solve the pendulum, which of the following is the best estimate for $z(3.1 \text{ s})$?

(A) 0.39 m

(B) 0.40 m

(C) 0.41 m

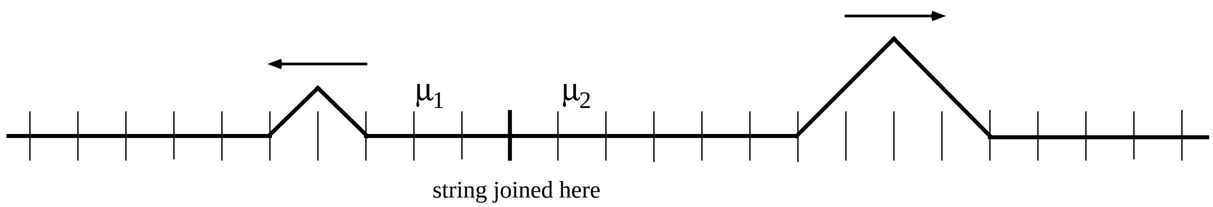
(D) 0.42 m

(E) 0.43 m

Answer

MC 4 **Reflection and Transmission at a Discontinuity** **[5 points]**

A pulse of height A_I, width X_I travels in a string of mass density μ_1 and is incident on a string of mass density μ_2. The pulse interacts with the junction between the two strings and a snapshot of the string a time later looks like the picture below (drawn to scale)



What is the ratio of the string mass densities, i.e., $\frac{\mu_1}{\mu_2}$?

(A) $\frac{\mu_1}{\mu_2} = \frac{1}{2}$

(B) $\frac{\mu_1}{\mu_2} = \frac{2}{1}$

(C) $\frac{\mu_1}{\mu_2} = \frac{4}{1}$

(D) $\frac{\mu_1}{\mu_2} = \frac{1}{4}$

(E) $\frac{\mu_1}{\mu_2} = \frac{1}{1}$

Answer

MC 5 **Standing Waves on a Guitar String** **[5 points]**

Elvis notices that his A string on his guitar is off pitch : it is vibrating at 550 Hz. He wants his A string to sound at 440 Hz.

He changes the tension. What is the ratio of the new to old tension, i.e., $\frac{\tau_{new}}{\tau_{old}}$?

(A) $\frac{\tau_{new}}{\tau_{old}} = \frac{4}{5}$

(B) $\frac{\tau_{new}}{\tau_{old}} = \frac{5}{4}$

(C) $\frac{\tau_{new}}{\tau_{old}} = \frac{25}{16}$

(D) $\frac{\tau_{new}}{\tau_{old}} = \frac{16}{25}$

(E) none of the above

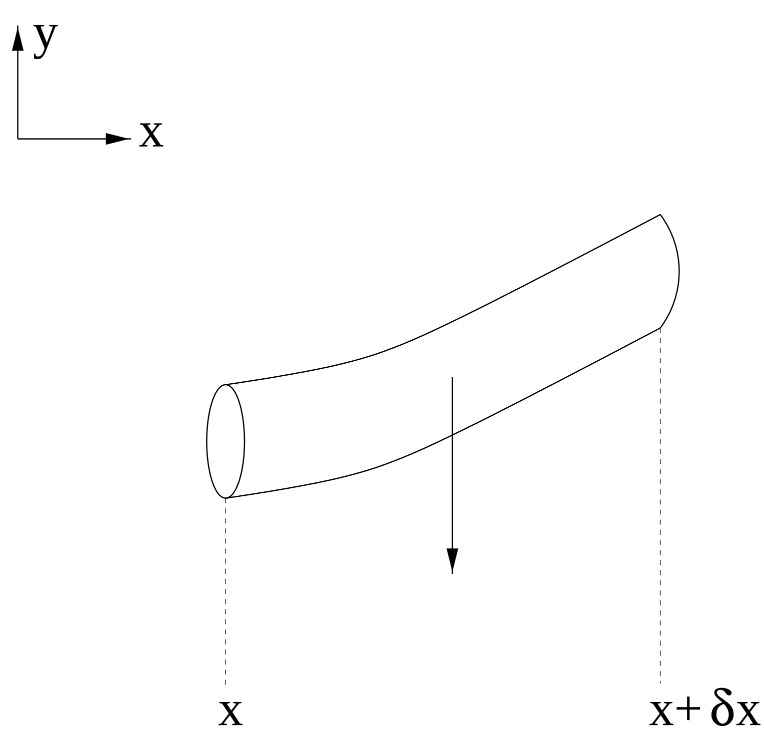
Answer

SA 1 **Deriving a Wave Equation** **[20 points]**

A rope of mass per unit length μ is stretched out horizontally with tension τ and is set into transverse motion in a viscous medium. We wish to determine the wave equation satisfied by the transverse displacement of the rope. Each chunk of the rope experiences a drag force, which is proportional to both the chunk length δx and the cube of the chunk (transverse) velocity $\dfrac{\partial y}{\partial t}$, with proportionality constant β. Note that the direction of the drag force is to slow down the chunk, acting in the opposite direction to the chunk (transverse) velocity so that

$$F_{drag} = -\beta \left(\frac{\partial y}{\partial t}\right)^3 \delta x \, .$$

(a) Draw on the diagram provided all the forces acting on the chunk of rope, i.e., a free body diagram. Note : **neglect gravity**. Resolve each force in to its horizontal (x) and vertical (y) components.



(b) Assuming that the motion of the chunk is purely transverse (vertical), what is the sum of the vertical components of the forces on the chunk in terms of $\tau, \beta, \delta x, y(x,t)$ and derivatives of $y(x,t)$?

$\sum F_y =$ ______________________________

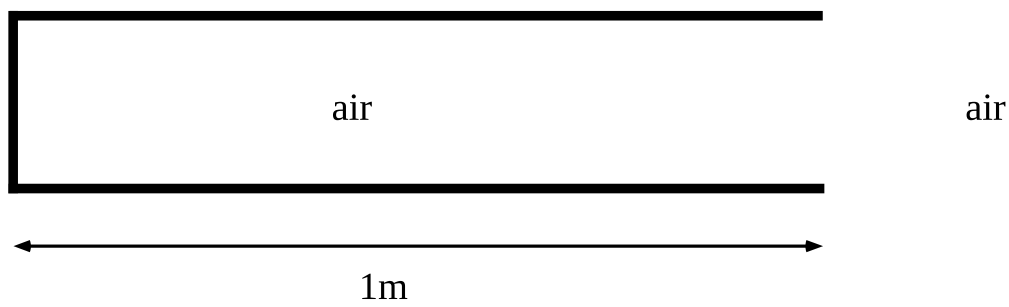
Note: No θs or F_{drag} symbols in your answer!

(c) Hence, using Newton's 2nd Law ($F = ma$), derive the wave equation for the transverse displacement $y(x, t)$ of the rope.

$$\frac{\partial^2 y}{\partial t^2} = \underline{\hspace{10cm}}$$

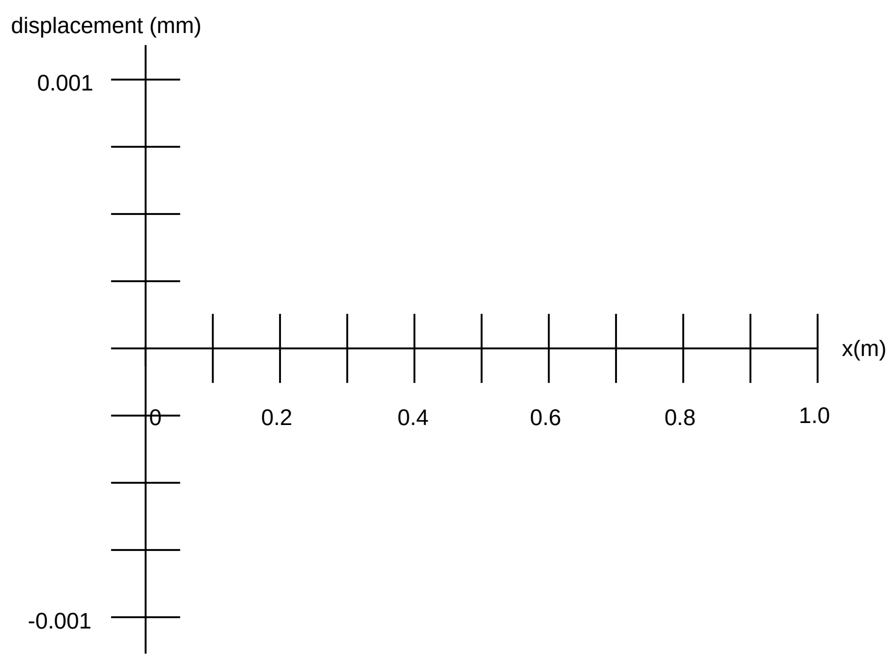
SA 2 **Sound Waves in a Pipe** **[25 points]**

A one meter long hollow plastic pipe filled with air is hanging horizontally in the air. At the left end, $x = 0$, the pipe is closed (no longitudinal motion possible in the air column at this position). At the right end, $x = 1$ m, the air column is open to the surrounding air (which corresponds to a free boundary condition).



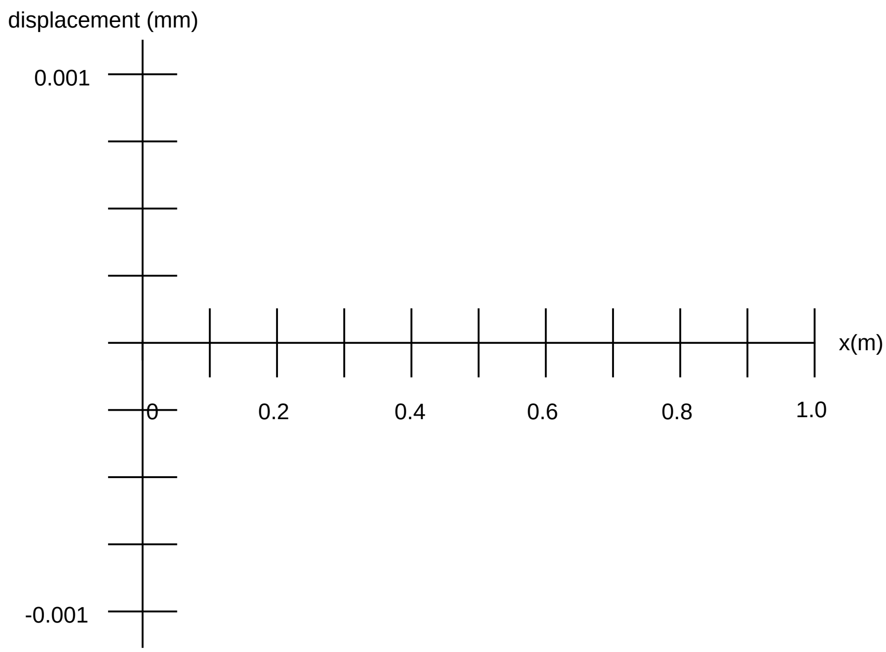
An engineer hears a tone at frequency $f_1 = 90$ cycles/s, which she determines is the fundamental standing wave frequency of the air within the pipe corresponding to the boundary conditions described above.

(a) Plot on the graph provided the horizontal displacement of the air column $s(x)$ versus x for the fundamental mode. Assume a maximum displacement of 0.001 mm.



Does your plot satisfy both boundary conditions?

(b) The engineer now places a microphone at the free end of the air column to detect the higher frequency standing waves in the air column. Plot on the graph provided the horizontal displacement of the air column $s(x)$ versus x for the next resonance above the fundamental mode. Assume again a maximum displacement of 0.001 mm.



(c) In the interior (not at the ends) region of the pipe's air column, where would a small floating particle wiggle the ***least*** under conditions described in part (b) of this problem?

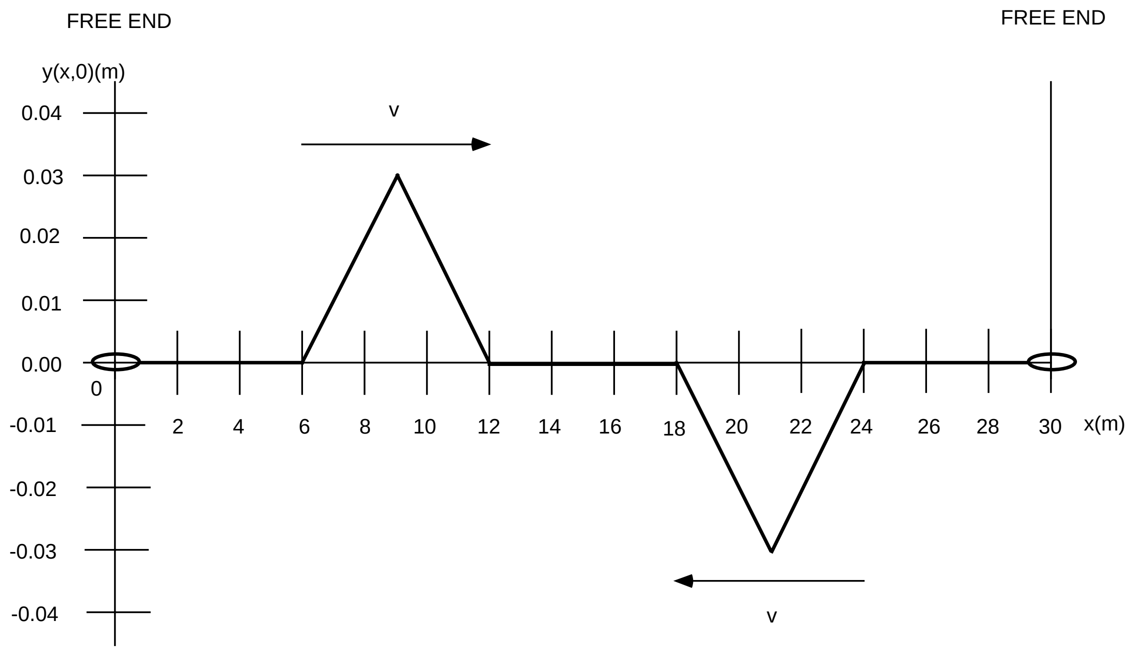
$x =$ ________________ m

(d) What is the numerical value of the frequency of the resonant mode described in part (b) of the is problem?

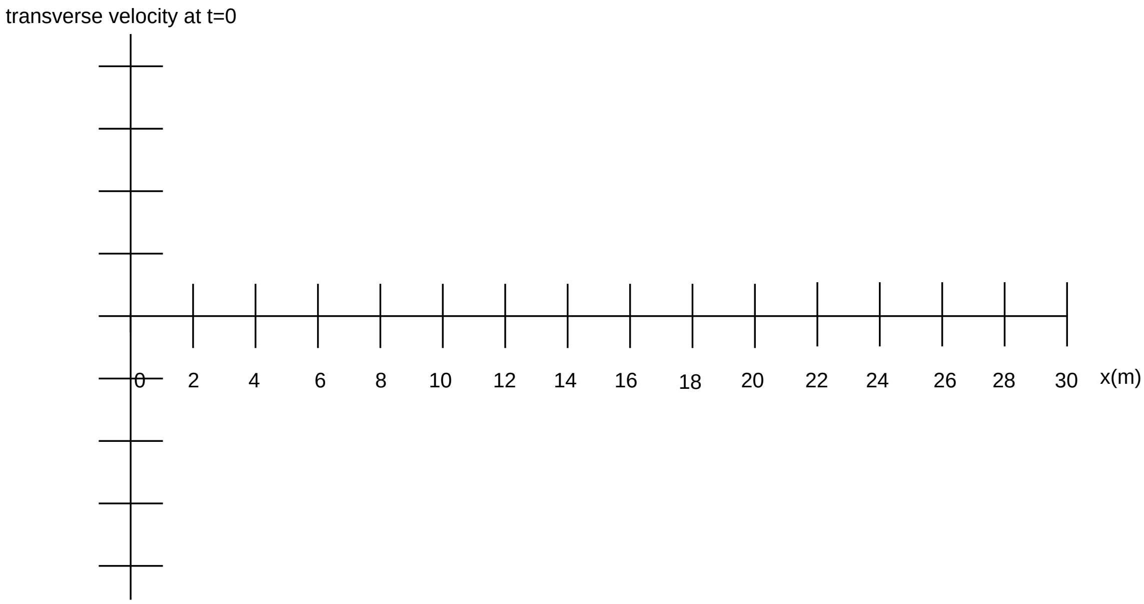
$f_1 = 90$ cycles/s ; $f_2 =$ ____________ cycles/s

SA 3 **Traveling Wave on a String** **[30 points]**

The figure below shows **two** traveling waves propagating on a string at time t=0. One wave is propagating to the right (positive x-direction) and the other to the left. The tension in the string is $\tau = 9$ N, and the string has a mass per unit length of $\mu = 1$ kg/m. The string has a length of 30 m, and has a free end at both ends, i.e., $x = 0$ m and $x = 30$ m.

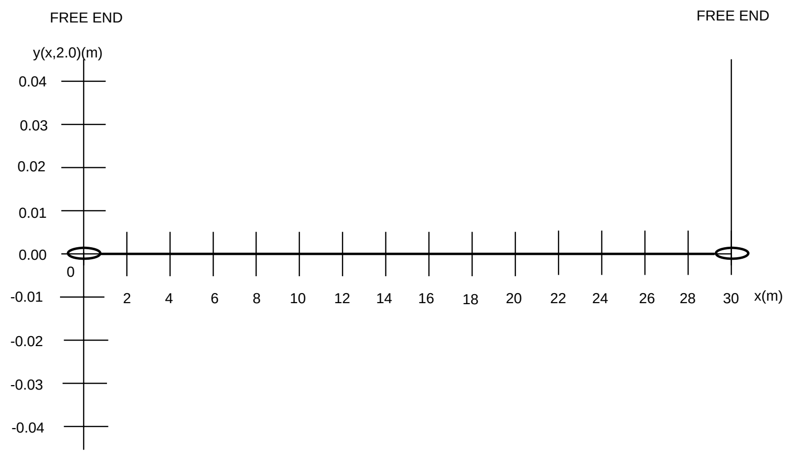


(a) Draw a graph of the transverse velocity (chunk velocity) of the wave at $t=0$, as a function of x.



Label your axes clearly and be sure to give units.

(b) At $t = 2.0$ s the string is perfectly flat.



The following is a statement from a student on why the string does not move after $t = 2.0$ s.

> *... since the spatial second derivative is zero, i.e.,* $\dfrac{\partial^2 y}{\partial x^2} = 0$ *and by the wave equation so must be the transverse (chunk) acceleration, i.e.,* $\dfrac{\partial^2 y}{\partial t^2} = 0$*. Also the pulses were traveling waves and since the string is flat then* $\dfrac{\partial y}{\partial x} = 0$ *and hence the transverse (chunk) velocity is also zero, i.e.,* $\dfrac{\partial y}{\partial t} = 0$*. Thus both the transverse (chunk) velocity and acceleration are zero and hence the string does not move.*

Tick in the boxes provided below each of the following student's statements which are **FALSE**.

$$\frac{\partial^2 y}{\partial x^2} = 0 \qquad \square$$
$$\frac{\partial^2 y}{\partial t^2} = 0 \qquad \square$$
$$\frac{\partial y}{\partial x} = 0 \qquad \square$$
$$\frac{\partial y}{\partial t} = 0 \qquad \square$$

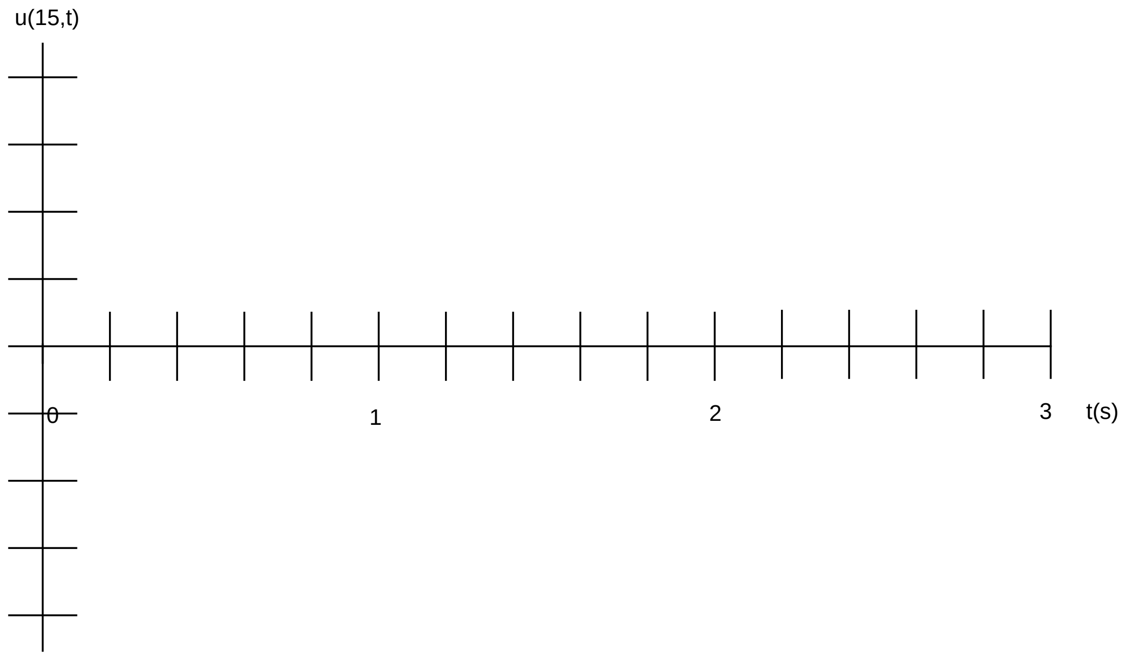
(c) Derive a expression for the power at each end of the string as a function of time t, i.e., $P(0, t)$ and $P(30, t)$.

$P(0, t) =$ ________________________________

$P(30, t) =$ ________________________________

PLEASE TURN PAGE

(d) Draw a graph of the energy density at the point $x = 15$ m as a function of time for $0 \leq t \leq 3$ s.



Label your axes clearly and be sure to give units.

END OF EXAM