Physics 214—Exam I (February 27) **Spring 1997**

NAME:______________________________

TA:__________________ **Section:**__________________

Multiple Choice (35 pts):_____ **x 5pts =** ______________

Short Answer: **S1 (15 pts)** ______________

S2 (30 pts) ______________

S3 (20 pts) ______________

TOTAL ______________

Multiple Choice: Be sure to put answers in boxes provided

M1. (5 pts) Sinusoids and radians.

In a sinusoidal traveling wave, the distance between two points that differ in phase by 2π radians is the

Answer

(A) frequency.

(B) amplitude.

(C) phase constant.

(D) wavelength.

(E) angular frequency.

M2. (5 pts) Velocity of packets.

A wave on a string has the form

$$y(x,t) = 0.01 \sin(10 \exp(-x^2 - 2xt - t^2))$$

where x and y are in meters and t in seconds.

In what direction is the pulse traveling (right is towards positive x), and with what speed?

Answer

$\boxed{}$

(A) Left, at ten meters per second.

(B) Right, at 0.01 meters per second.

(C) Left, at two meters per second.

(D) Right, at two meters per second.

(E) Left, at one meter per second.

(F) Right, at one meter per second.

M3. (5 pts) Traveling sine wave.

A wave on a string has transverse displacement

$$y(x,t) = 3\sin(6x - 4t),$$

where x is in meters, and t in seconds. Which one is true of this wave?

Answer

(A) The frequency f is 4 cycles per second.

(B) The frequency f is $2/\pi$ cycles per second.

(C) The frequency f is 8π cycles per second.

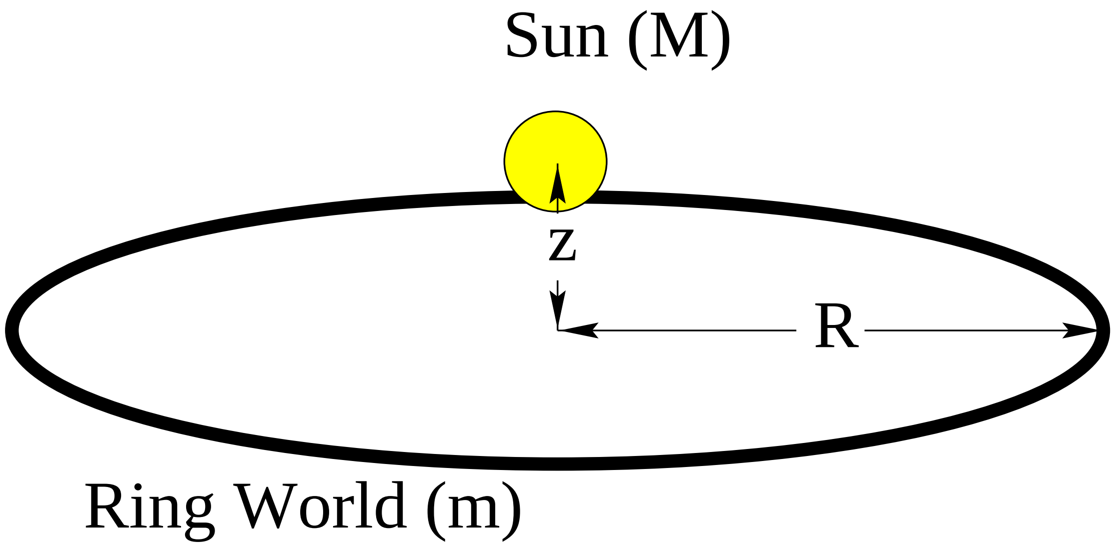
(D) The frequency f is $3/\pi$ cycles per second.

(E) The frequency f is 6 cycles per second.

(F) None of the above is true.

M4. (5 pts) Ringworld.

An advanced alien civilization redesigns their planet of mass m in the shape of a giant ring of radius R, with the sun (mass M) at the center.



If the ring is displaced a distance z perpendicular to the plane of the ring, one can use Newton's universal law of gravitation to write the force on the ring in terms of the gravitational constant G. For small z and $m << M$, the ring accelerates according to the equation

$$\frac{d^2z}{dt^2} = -\frac{GMz}{R^2}$$

What is the frequency (ω, in radians per second), of small oscillations of the ring-world about $z = 0$?

Answer

(A) $\omega = \sqrt{G/R}$

(B) $\omega = \sqrt{\frac{GM}{mR^2}}$

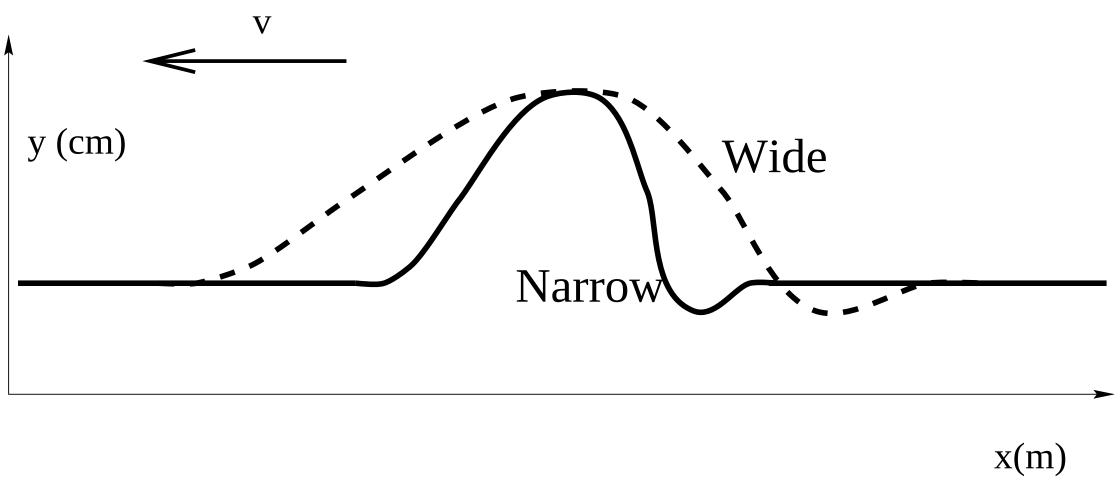
(C) $\omega = \frac{1}{2\pi}\sqrt{\frac{GM}{z^2+R^2} + \frac{2GMz^2}{(z^2+R^2)^2}}\Big|_{z=2\pi R}$

(D) $\omega = \sqrt{2\pi R\tau m}$

(E) $\omega = \sqrt{GM/R^2}$

M5. (5 pts) Pulse Energies and Scales.

What is the energy of the wide pulse, compared to that of the narrow pulse? The wide one is twice as wide in the x direction, but exactly the same shape in the y direction, and moves at the same speed on the same string.



Answer

$\boxed{}$

(A) $E_{wide}/E_{narrow} = 4$

(B) $E_{wide}/E_{narrow} = 2$

(C) $E_{wide}/E_{narrow} = \sqrt{2}$

(D) $E_{wide}/E_{narrow} = 1$

(E) $E_{wide}/E_{narrow} = 1/2$

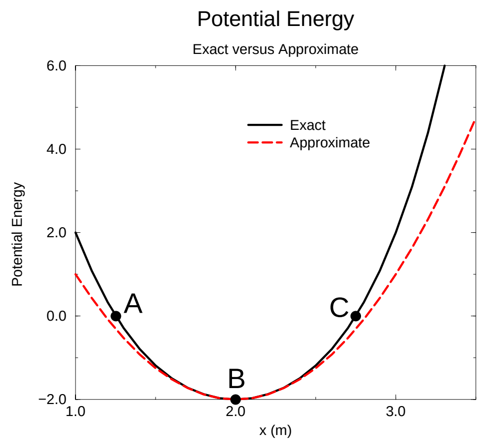
M6. (5 pts) Exact vs. Approximate Frequencies.

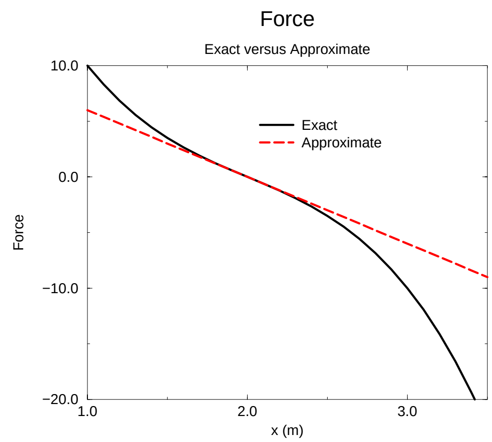
A mass M is attached to the end of a massless, nonlinear spring. The energy and force as a function of the length x of the spring are as plotted below. The exact equation of motion for the mass is

$$M\ddot{x} = -6(x-2) - 4(x-2)^3$$

An approximate equation of motion is given by keeping only the first term:

$$M\ddot{x}_{approx} = -6(x-2)$$





Which one is true?

Answer

(A) The exact system will undergo stable, small oscillations about point A, with frequency higher than that of the approximate equation of motion.

(B) The exact system will undergo stable, small oscillations about point A, with frequency lower than that of the approximate equation of motion.

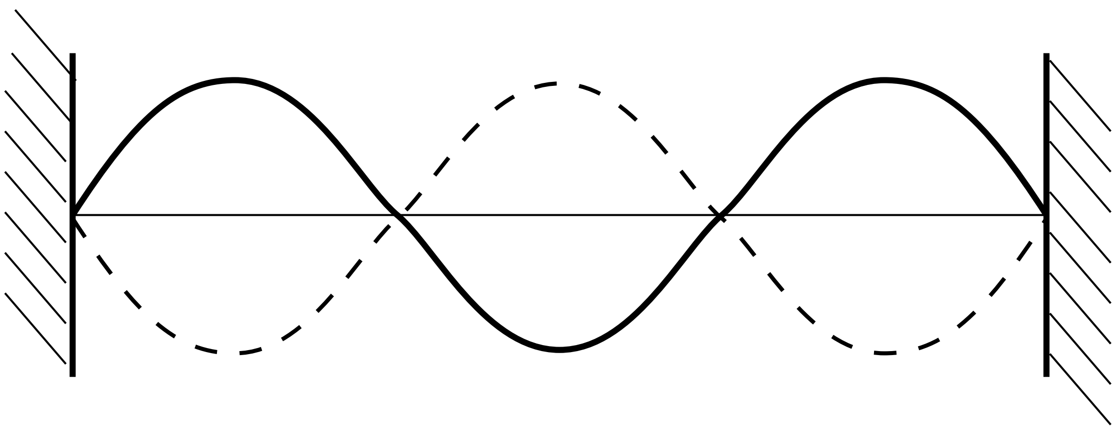
(C) The exact system will undergo stable, small oscillations about point B, with frequency higher than that of the approximate equation of motion.

(D) The exact system will undergo stable, small oscillations about point B, with frequency lower than that of the approximate equation of motion.

(E) The exact system will undergo stable, small oscillations about point C, with frequency lower than that of the approximate equation of motion.

M7. (5 pts) Standing Waves

A stretched string of mass density $\mu = 0.01$ kg/m, tension 1 N, and length 3 m, is vibrating in the standing-wave pattern shown. What is f, the frequency of oscillation, in cycles per second?



Answer

$\boxed{}$

(A) $f = 5$ Hz.

(B) $f = 10$ Hz.

(C) $f = 3.33$ Hz.

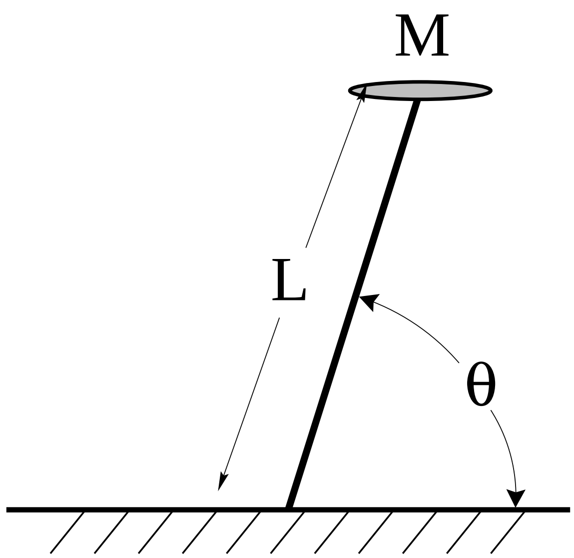
(D) $f = 10/\pi$ Hz.

(E) $f = 20\pi/3$ Hz.

Short Answer: Show Your Work

S1. (15 pts) Small Angles, Taylor's Theorem, and Unstable Equilibria.

Marj the juggler has balanced a dinner plate of mass M at the end of a massless pole of length L. Treat the dinner plate as a point mass attached to the end of the pole.



In this problem, we will explore the behavior of the pole and plate, treating it as a simple, undamped pendulum, for angles near the *unstable* equilibrium. We remind you that the equation of motion for the angle of the pole with respect to the ground (*not* the angle with respect to the vertical!) is

$$\ddot{\theta} = -(g/\ell)\ \cos\theta$$

(A) (5 points)
What is the smallest, positive value of θ for which the pole has an *unstable* equilibrium θ_U? (No calculation required! Think physically.) Draw a picture of the pole at the unstable point.

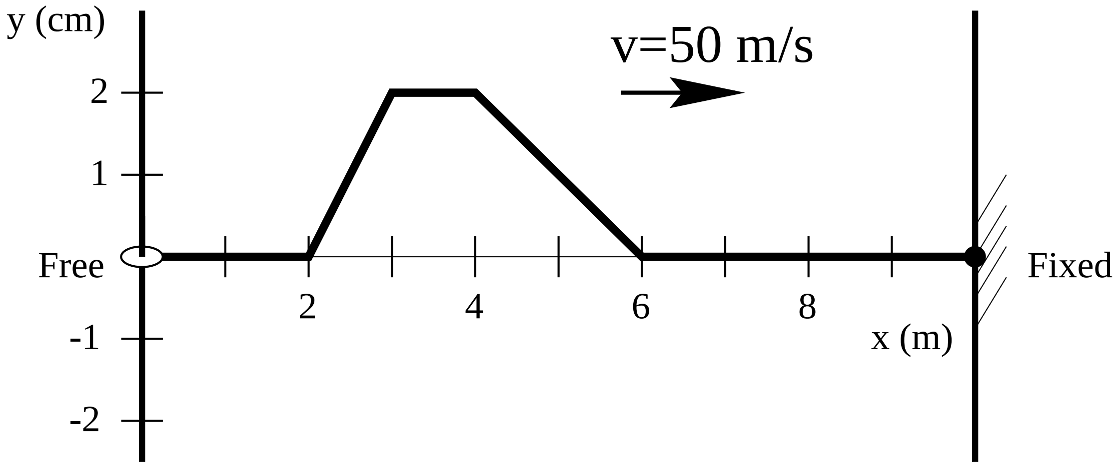
$\theta_U =$ ______________________

(Did you remember that θ is the angle from the *horizontal*?)

(B) (10 points)
Derive an approximate equation of motion for the pendulum, accurate to first order in the angular deviation $\delta = (\theta - \pi/2)$, by expanding $\cos(\pi/2 + \delta)$ in a Taylor's expansion about $\delta = 0$. Write your answer in terms of θ, not δ.

$\ddot{\theta} =$ ________________________

S2. (30 pts) Propagating Pulses.

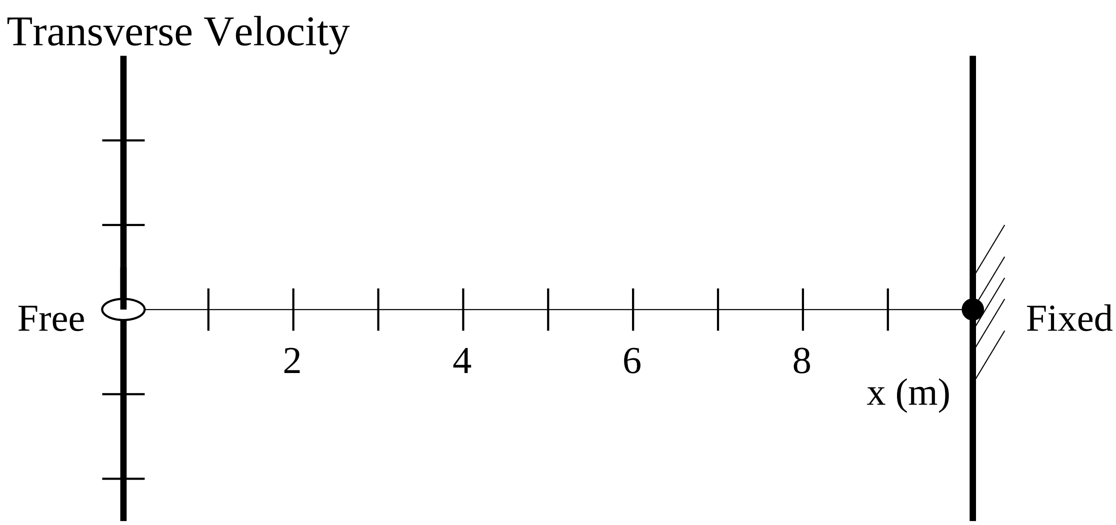


A stretched cable of mass density $\mu = 4$ kg/m has a free boundary condition at $x = 0$ and a fixed boundary condition at $x = 10$m. An asymmetric pulse is traveling in the $+x$ direction at a velocity $v = 50$ m/s. At $t = 3$ seconds the pulse is 2cm high, and shaped as shown in the graph above.

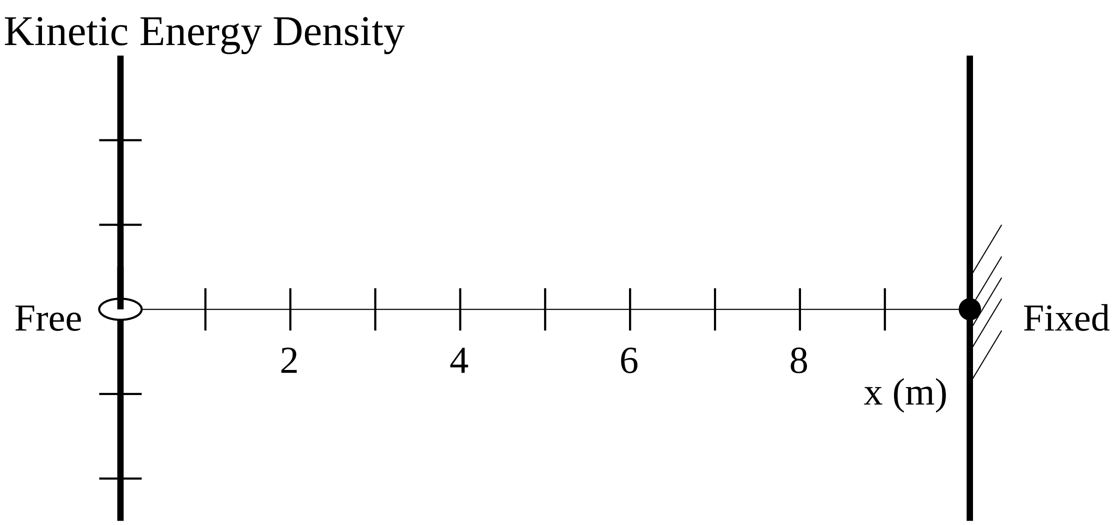
(A) (5 points) What is the tension τ in the string?

$\tau =$____________

(B) (5 points) Graph the transverse velocity $\partial y/\partial t$ of the string at the time $t = 3$ seconds shown in the graph above. Label the values of the tick-marks on the vertical axis (including units)!

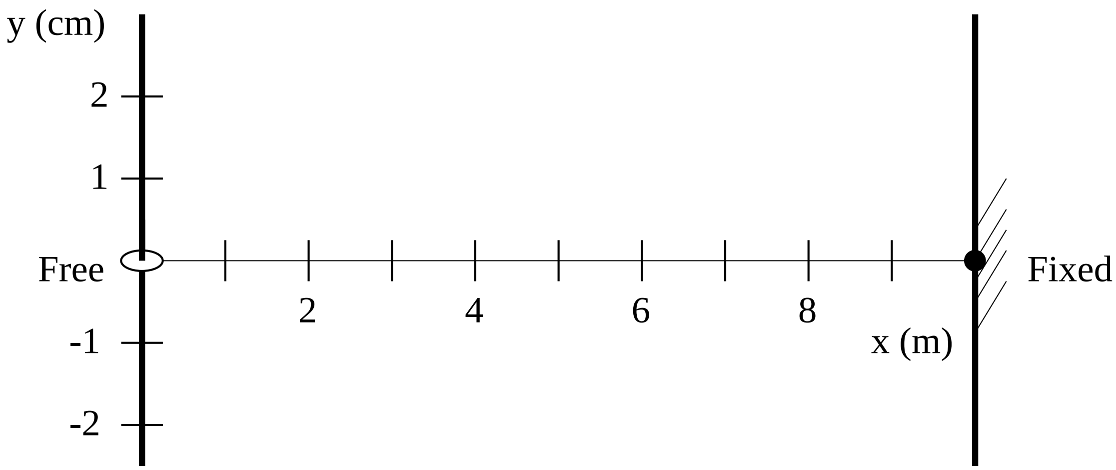


(C) (5 points) Graph the *kinetic* energy density $u(x, t)$ of the string at the time $t = 3$ seconds shown in the graph above. Label the values of the tick-marks on the vertical axis (including units)!



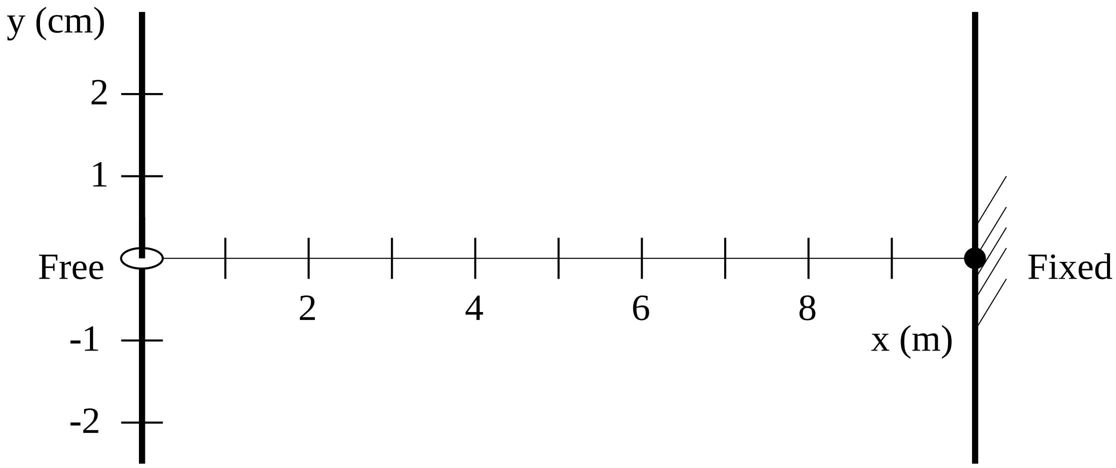
(Did you remember to include the units for the vertical axis?)

(D) (5 points) Draw the pulse as it was in the past at $t = 2.8$ seconds. Draw an arrow above the pulse showing the direction that the pulse is moving at $t = 2.8$ seconds.



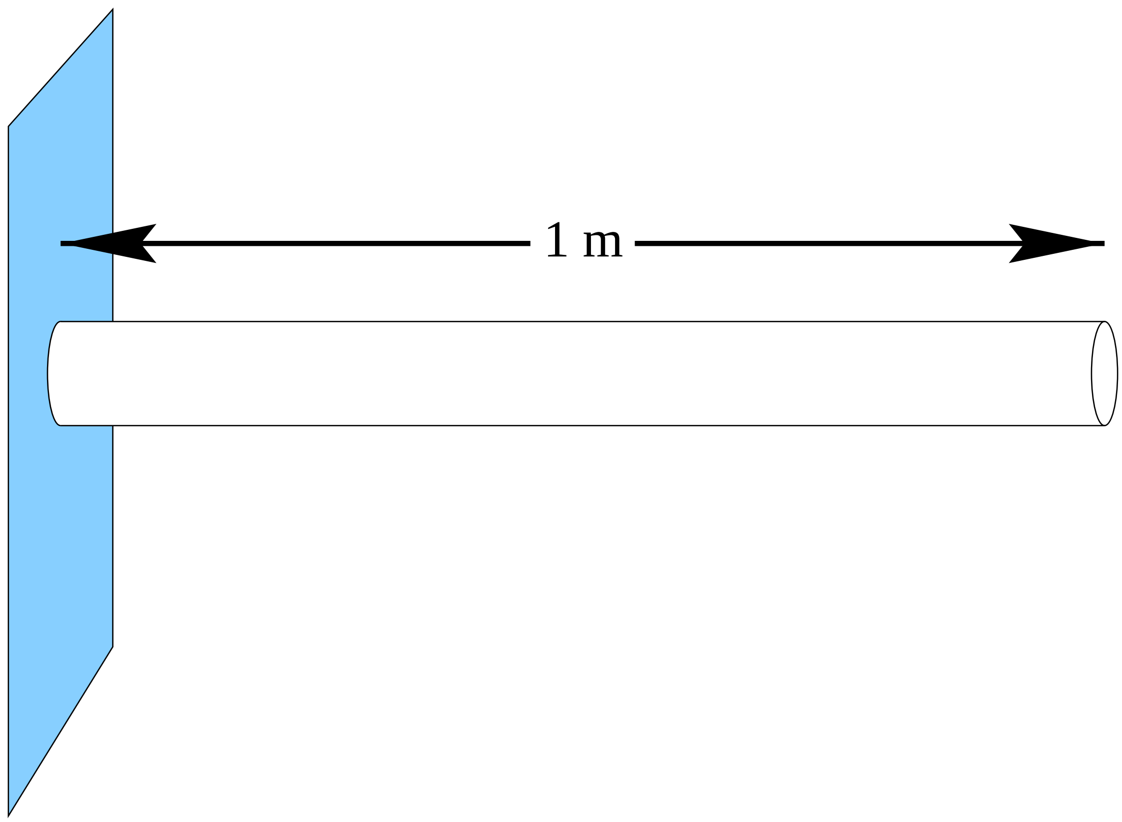
(This problem is independent of parts (ABC). Did you use the boundary condition at $x = 0$? Did you remember to draw the arrow for the velocity?)

(E) (10 points) Draw the pulse as it will be at $t = 3.12$ seconds.



(This problem is independent of parts (ABCD).)

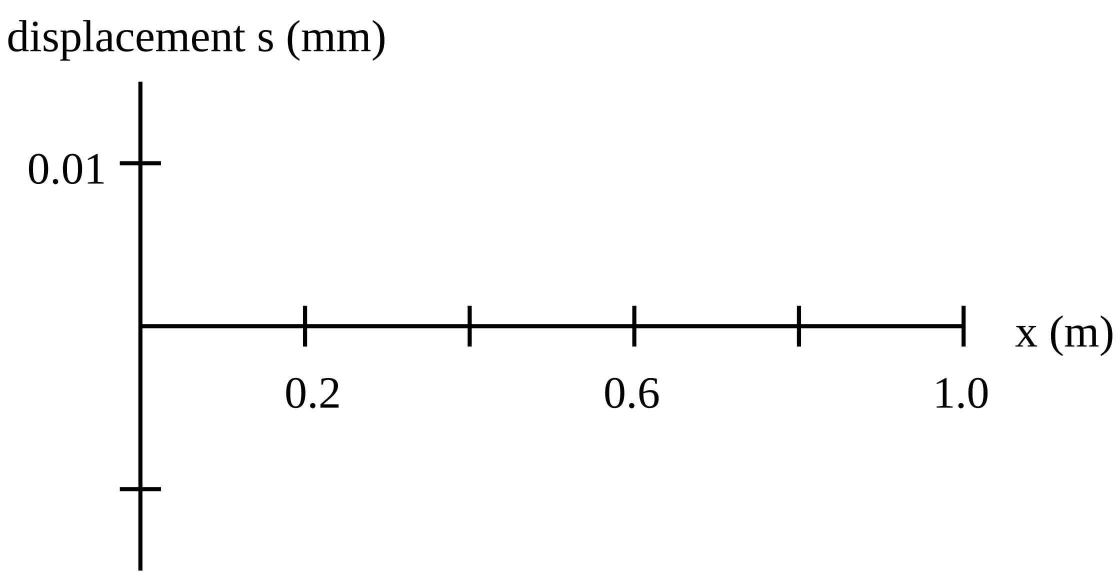
S3. (20 pts) Aluminum Rod.



A one meter aluminum rod is hanging horizontally in the air. On the left at $x = 0$, it is rigidly clamped at the wall (no longitudinal motion possible at $x = 0$). On the right at $x = 1$ m, it is hanging freely in the air (as were both ends of the rod in Experimental Lab I).

An engineer hears a tone from the rod at $f_1 = 1250$ Hz, which she figures out is the fundamental standing-wave frequency of the rod with boundary conditions as discussed above.

(A) (5 points) Graph the horizontal displacement of the rod $s(x)$ versus x for the fundamental mode. Assume a maximum displacement of 0.01mm.



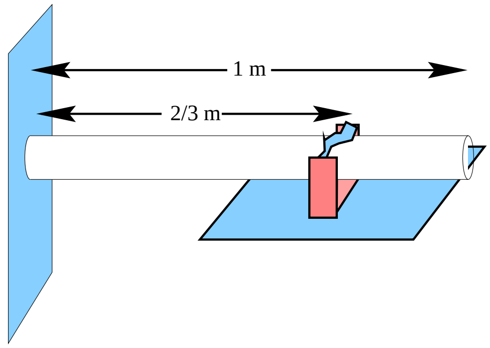
(B) (5 points) She then attaches a piezoelectric transducer to the free end of the rod and looks for higher frequency standing waves. What will be the next two resonant frequencies f_2 and f_3 above the fundamental?

$f_1 = 1250$ Hz

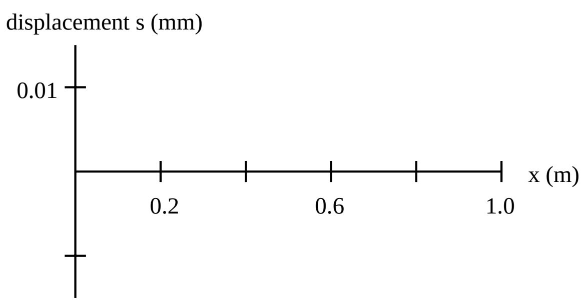
$f_2 =$ ____________

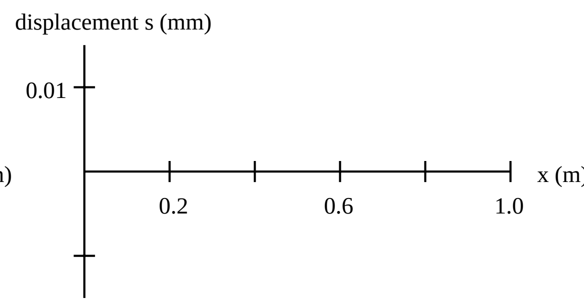
$f_3 =$ ____________

(C) (10 points) The rod is now gently clamped at a distance 2/3 m from the wall. The clamp looks like the one in the middle of the rod for experimental lab #1, except that it is screwed down more firmly.



Which of the three standing waves (frequencies f_1, f_2, or f_3 in part (B)) will the experimentalist find is damped the *least*? Draw graphs of the longitudinal displacements of the two next modes.





Circle one: f_1 f_2 f_3

Physics 214—Exam I (February 27) Spring 1997
Formula Sheet

Particle in a potential well. For a particle of mass m in a potential $U(x)$, the force $F = -\frac{dU}{dx}$. Near a point x_0 where the force is zero, the small oscillation frequency ω, in radians per second, obeys $\omega^2 = -\frac{1}{m}\left.\frac{dF}{dx}\right|_{x_0}$

Math Stuff. Taylor's theorem says that a function f near a point x_0 can be expanded in powers of $(x - x_0)$ as follows:

$$f(x) \approx f(x_0) + (x - x_0)\left.\frac{df}{dx}\right|_{x_0} + \frac{(x-x_0)^2}{2!}\left.\frac{d^2 f}{dx^2}\right|_{x_0} + \ldots + \frac{(x-x_0)^n}{n!}\left.\frac{d^n f}{dx^n}\right|_{x_0} + \ldots$$

The angle addition formulas are $\sin(A+B) = \sin A \cos B + \cos A \sin B$ and $\cos(A+B) = \cos A \cos B - \sin A \sin B$. For a sine wave, $f = \omega/2\pi$, and $k = 2\pi/\lambda$. As a crude approximation, the n^{th} derivative $d^n y/dx^n \approx A/\Delta x^n$.

Wave Equations. The wave equation

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

has a traveling wave solution $y(x,t) = f(x \pm vt)$, a standing-wave solution $y(x,t) = A\sin(kx)\sin(\omega t)$, and (as a special case) a traveling sine wave $y(x,t) = A\sin(kx - \omega t)$, where $\omega/k = v$.

For transverse waves on a string, the velocity of a traveling wave solution is $v = \pm\sqrt{\tau/\mu}$, the energy density (kinetic energy density + potential energy density) $u(x,t) = \frac{\mu}{2}\left(\frac{\partial y}{\partial t}\right)^2 + \frac{\tau}{2}\left(\frac{\partial y}{\partial x}\right)^2$, and the power $P(x,t) = -\tau \frac{\partial y}{\partial t}\frac{\partial y}{\partial x}$.

For transverse *traveling* waves on a string, of the form $y(x,t) = f(x \pm vt)$, $\frac{\partial y}{\partial x} = \pm\frac{1}{v}\frac{\partial y}{\partial t}$. Also, the energy density can be simplified to $u(x,t) = \tau\left(\frac{\partial y}{\partial x}\right)^2$ for such traveling waves.

For sound waves in gasses and liquids, the velocity of sound is $\sqrt{B/\rho}$. The velocity of sound in air is about 340 m/s.

Fixed boundary conditions and free boundary conditions for standing waves have $f_n = nv/2L$; mixed boundary conditions have $f_n = (2n-1)v/4L$.