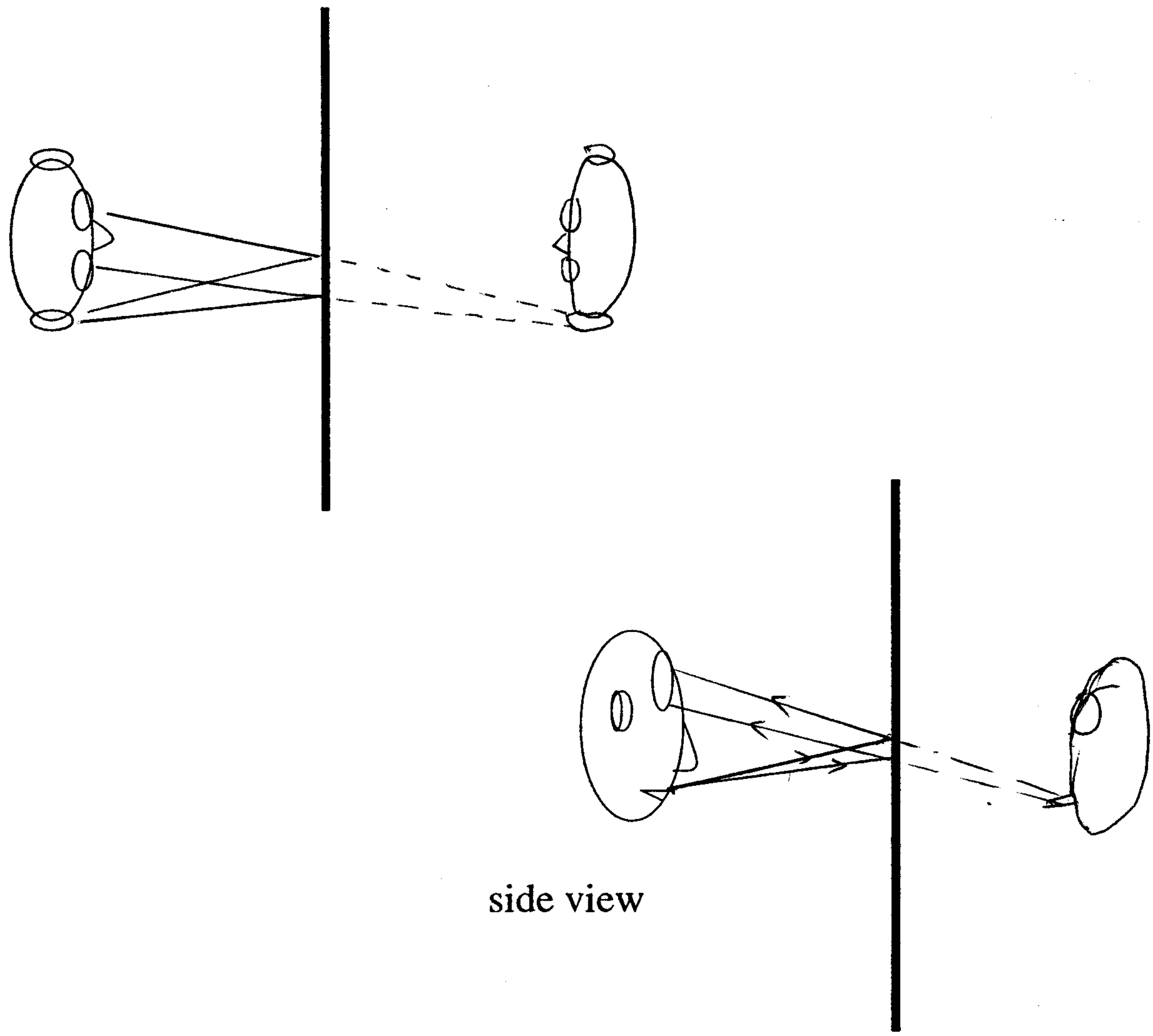
# 1 Mirror Reflection [10 points]

Reflection at a mirror: When you look into a mirror, do you see your face reversed ? Show the light rays in each of the diagrams why this is so.

top view



side view

Figure 1: Your face in front of the mirror.

Is it left-right reversed ? Yes or No. (Hint: Think of the letter **F** and its image.)

NO

Is it up-down reversed ? Yes or No.

NO

## 2 Geometric Optics [20 points]

A thin biconvex lens of focal length $f_1$ is imaging an object of height H at distance $p$ away from the lens.

Plot the necessary principle rays to show where the image is.

Express the distance $q$ of the image from the lens in terms of $f_1$ and $p$.

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \qquad \frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{p-f}{pf}$$

$$\boxed{q=\frac{pf}{p-f}}$$

What is the height $h$ of the image? Express it in turns of $p$, H and $q$.

$$\boxed{h=-\frac{q}{p}H}$$

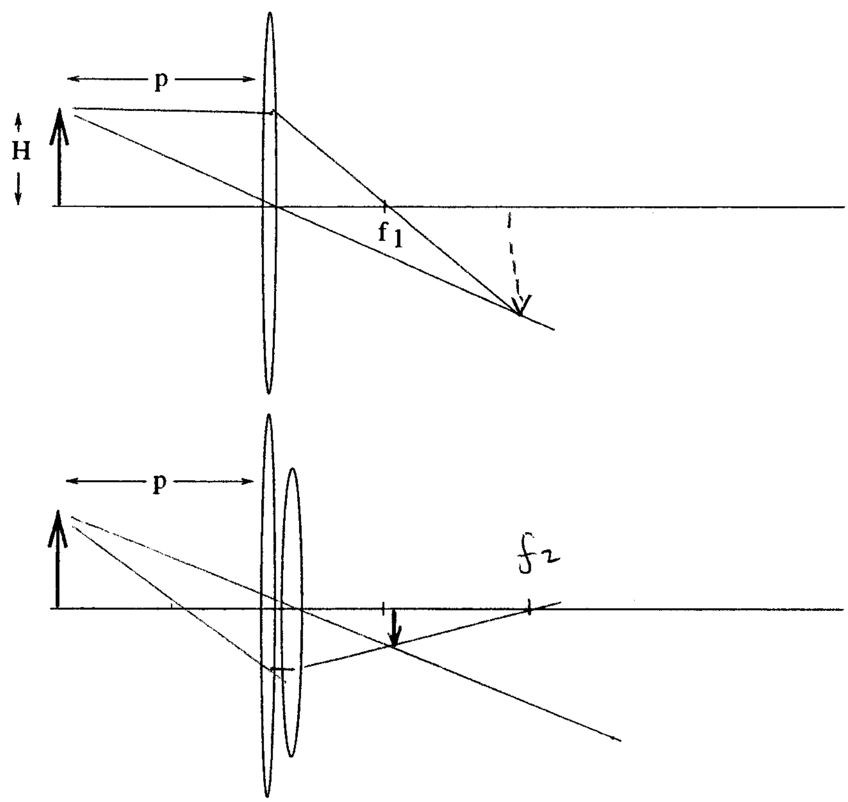



Figure 2: Object and Image

Now a second thin biconvex lens of focal length $f_2$ is placed next to the first lens,
Plot the necessary principle rays to show where the image is.

How far is the new image from the lens ? Use the thin lens approximation and ignore the separation between the lenses.

$$\boxed{q = \frac{p f_1 f_2}{p(f_1+f_2) - f_1 f_2}}$$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}$$

$$q = \frac{pf}{p-f} = \frac{p f_1 f_2}{p(f_1+f_2) - f_1 f_2}$$

The setup below is similar to that in the lab you did, involving a thin biconvex lens with focal length $f_C = 10$ cm, and a thin diverging lens with focal length $f_D = -20$ cm. Locate the image (as the distance to the right of the diverging lens).

$$\boxed{60 \text{ cm.}}$$

$$\frac{1}{p} + \frac{1}{q_1} = \frac{1}{f_C}$$

$$\frac{1}{q_1} = \frac{1}{10} - \frac{1}{15} = \frac{1}{30} \qquad q_1 = 30$$

$$-\frac{1}{q_1 - 15} + \frac{1}{q} = \frac{1}{f_D} = -\frac{1}{20}$$

$$\frac{1}{q} = -\frac{1}{20} + \frac{1}{15} = \frac{1}{60}$$

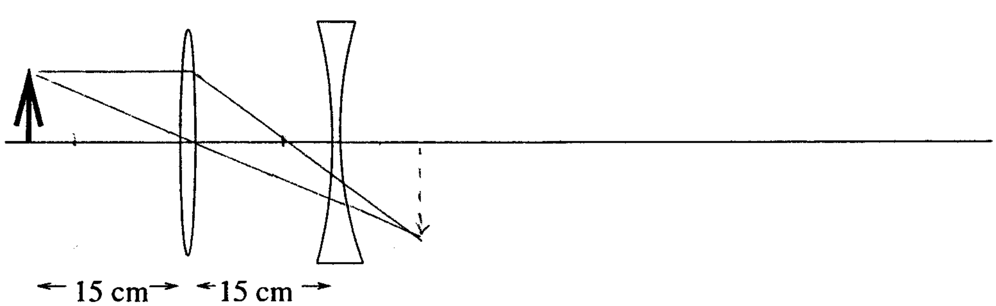



Figure 3: A Biconvex Lens and A Diverging Lens

# 3 Sound Wave [15 points]

Consider sound wave in air (with air density $\rho$). The displacement is given by

$$s(x) = Ae^{i(kx-\omega t)} \tag{3.1}$$

What is the value of the bulk modulus $B$ in terms of $\rho$, $k$ and $\omega$ ?

$$\frac{\omega^2}{k^2} = c^2 = \frac{B}{\rho} \qquad B = \frac{\omega^2}{k^2}\rho$$

$$\boxed{B = \frac{\omega^2}{k^2}\rho}$$

The bulk modulus $B$ has the same dimensions as

(A) force
(B) pressure
(C) distance
(D) energy
(E) speed
(F) momentum
(G) acceleration

$\boxed{B}$

What is the maximum speed of a small element of air in the presence of the sound wave?

$$\frac{\partial s}{\partial t} = -i\omega A e^{i(kx-\omega t)}$$

$$\max\left|\frac{\partial s}{\partial t}\right| = \omega A$$

$\boxed{\omega A}$

# 4 Superposition of Transverse Waves [30 points]

Consider a transverse wave $z(x, y, t)$ on a thin flat stretched membrane.

Suppose the wave is described by

$$z(x, y, t) = Ae^{i(\mathbf{k}\cdot\mathbf{r}+wt)} \tag{4.1}$$

where $\mathbf{k} \cdot \mathbf{r} = k_1 x + k_2 y$, with $k_1 > 0$ and $k_2 > 0$.

What is the speed c ?

$$c^2 = \frac{\omega^2}{k_1^2 + k_2^2}$$

$$\boxed{\frac{\omega}{\sqrt{k_1^2+k_2^2}}}$$

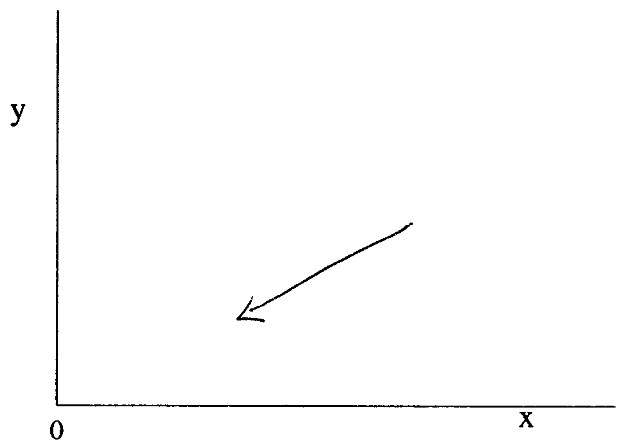



Figure 4: The stretched membrane in the $xy$-plane.

What is the direction of the wave ?

$$+\vec{k} = (k_1, k_2)$$

$$\boxed{(-k_1, -k_2)}$$

Point out the direction in the figure if $k_1 = 2k_2$.

The membrane lies in the upper-right quadrant ($x > 0$, $y > 0$) of the $xy$-plane, as shown.

The membrane has a fixed edge along the line $x = 0$; that is $z(x = 0, y, t) = 0$.

What is the reflected wave $z_r(x, y, t)$ ?

$$\boxed{-Ae^{i(-k_1 x + k_2 y + \omega t)}}$$

Write down the resulting superposition of the two waves. (Make sure you combine them.)

$$z + z_r = A e^{i(k_2 y + \omega t)} \left( e^{ik_1 x} - e^{-ik_1 x} \right)$$
$$= 2iA \sin(k_1 x)\, e^{i(k_2 y + \omega t)}$$

$$\boxed{2iA \sin(k_1 x)\, e^{i(k_2 y + \omega t)}}$$

Suppose there is also a fixed edge along the line $y = 0$; that is $z(x, y = 0, t) = 0$. (So both edges are fixed.) Write down the resulting wave in this case.

$z = 0$ at $x = 0$ $\forall t$

$z = 0$ at $y = 0$ $\forall t$

$$z = 2iA \sin(k_1 x)\, e^{i\omega t} \left[ e^{ik_2 y} - e^{-ik_2 y} \right]$$

$$\boxed{-4A \sin k_1 x \sin k_2 y\, e^{i\omega t}}$$

Where are the nodal lines (lines along which z=0 at all t) ?

$k_1 x = n\pi$ $\quad x = \frac{n\pi}{k_1}$

$k_2 y = m\pi$ $\quad y = \frac{m\pi}{k_2}$

$$\boxed{x = \frac{n\pi}{k_1} \quad n = 0, 1, 2, \ldots \qquad y = \frac{m\pi}{k_2} \quad m = 0, 1, 2, \ldots}$$

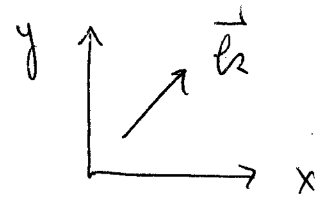
## 5 EM Waves [25 points]

The electric field of an EM wave is described by

$$\mathbf{E}_1(x, y, z, t) = \hat{\mathbf{z}} E_0 e^{i(k_1 x + k_1 y - \omega t)}. \tag{5.1}$$

where $E_0$, $k_1 > 0$, $\omega$ are real. Here, $\mathbf{k} = (k_1, k_1, 0)$.

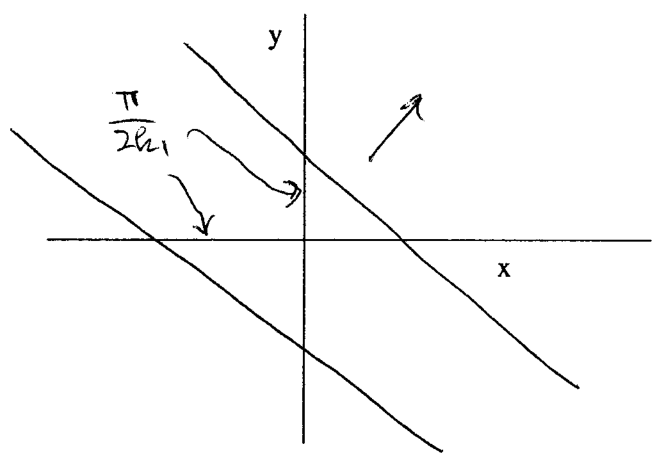
Which direction is the wave traveling?




$\hat{x} + \hat{y}$

$(1, 1, 0)$

At $t = 0$, draw the nodal line(s) closest to $(x, y) = (0, 0)$ in the $xy$-plane in the figure below.




$e^{ik_1(x+y)}$ is pure imaginary

$e^{i(k_1)(x+y)} = e^{\pm i\pi/2}$

$x + y = \pm \frac{\pi}{2k_1}$

Figure 5: The $xy$-plane.

Write down the magnetic field **B** corresponding to this wave.

$\vec{B} = (\hat{x} - \hat{y}) \frac{E_0}{\sqrt{2}\, c} e^{i(k_1 x + k_1 y - \omega t)}$

Suppose another EM wave with

$$\mathbf{E}_2(x,y,z,t) = (\hat{\mathbf{y}}+\hat{\mathbf{z}})E_0 e^{i(k'x-\omega t)} \tag{5.2}$$

crosses path with the above EM wave described by $\mathbf{E}_1$. Take $k' > 0$.

What is the value $\frac{k_1}{k'}$ ?

$$\frac{\omega}{\sqrt{k_1^2+k_1^2}} = \frac{\omega}{\sqrt{2}\,k_1} = c$$

$$\frac{\omega}{k'} = c \quad \text{so} \quad \frac{k_1}{k'} = \frac{1}{\sqrt{2}}$$

$$\boxed{\frac{1}{\sqrt{2}}}$$

Write down the resulting electric field coming from the superposition of these two waves?

$$\vec{E} = \hat{z}\, e^{i(k_1x+k_1y-\omega t)} E_0 + (\hat{y}+\hat{z})E_0 e^{i(k'x-\omega t)}$$

$$= \hat{z}E_0\left(e^{i(k_1x+k_1y)} + e^{ik'x}\right)e^{-i\omega t}$$

$$+\hat{y}E_0 e^{i(k'x-\omega t)}$$

END OF EXAM