P661 Homework # 1 8/26/04

Due Tuesday 9/7/04

Read Polchinski Ch 1.

Read Johnson Ch. 1 and earlier parts of Ch. 2.

Part 1

(1) Problem 1.2

(2) For the lightest spin-2 closed string mode to be massless, what is the spacetime dimension and what is the mass of the closed string tachyon ?

In this case, what other massless closed string modes are present in the oriented case ?

(3) Problem 1.4. Skip $\alpha' m^2 = 2$. How many modes are there at levels $\alpha' m^2 = 0, 1$?

(4) Consider the famous Dedekind η function

$$\eta(q) = q^{1/24} \Pi_{n=1}^{\infty} (1 - q^n) \tag{1}$$

Expanding $\eta(q)^{-24}$, we have

$$\eta(q)^{-24} = \sum p(N) q^N = \frac{1}{q} + 24 + p(1)q + p(2)q^2 + ... \tag{2}$$

Compare $p(0) = 24$ and $p(1)$ with the number of modes at level $\alpha' m^2 = 0, 1$, respectively.

Convince yourself that $p(N)$ is the number of modes at level N.

We shall come back to understand the meaning of this formula. Note that, asymptotically, $\ln p(N) \sim \sqrt{N}$.

Part 2:

(5) Do problems (3) and (4) for the level $\alpha' m^2 = 2$.

(6) At the classical level, check that the Nambu-Goto action yields the same first quantized spectrum as that of the Polyakov action.

(7) Problem 1.3.