P661 Homework # 5 09/23/04

Due Tuesday 10/05/04

Briefly read Chapter 3. A careful reading of this chapter is reserved for **Part II**.

Part 1

(1) Look at Eq.(3.5.6) (forget about c here). Consider a closed string N-point M-loop scattering amplitude. What is the Euler number χ ? If you shrink the string to a point, this amplitude reduces to a Feynman diagram. Show that $-\chi$ is the power of the 3-point coupling expected.

(2) Consider a 5-point 1-loop scattering process. How many Feynman diagrams are there ? Give an estimate. Consider pure Yang-Mills theory. If you are not familiar with it, consider $\lambda\phi^3$.

Part 2:

(3) Verify Eq.(2.5.12) for the case $\lambda = 2$. What is the central charge c for this bc CFT.

(4) Veneziano and Shapiro-Virasoro amplitudes.

Recall the identity:

$$|z|^{-a}\Gamma(a/2) = \int_0^\infty dt t^{a/2-1} e^{-t|z|^2} \tag{1}$$

We shall use this identity a few times here.

(a) Verify that Eq.(6.4.10) becomes Eq.(6.4.24) for $x = s$ and $y = t$.

(b) Verify Eq.(6.6.11). Hint: use the above identity twice, introducing t and u variables. (Now you have 2 Γ functions downstair.) Writing $z = x + iy$, the integrals over x and y become Gaussian. Now introduce new variables $t = wv$ and $u = (1-w)v$, with $0 < w < 1$ and $0 < v < \infty$. (What is the Jacobian ?) Integrating v gives you another Γ function and the remainng integration is similar to part (a).