Missing E_T Significance in ATLAS

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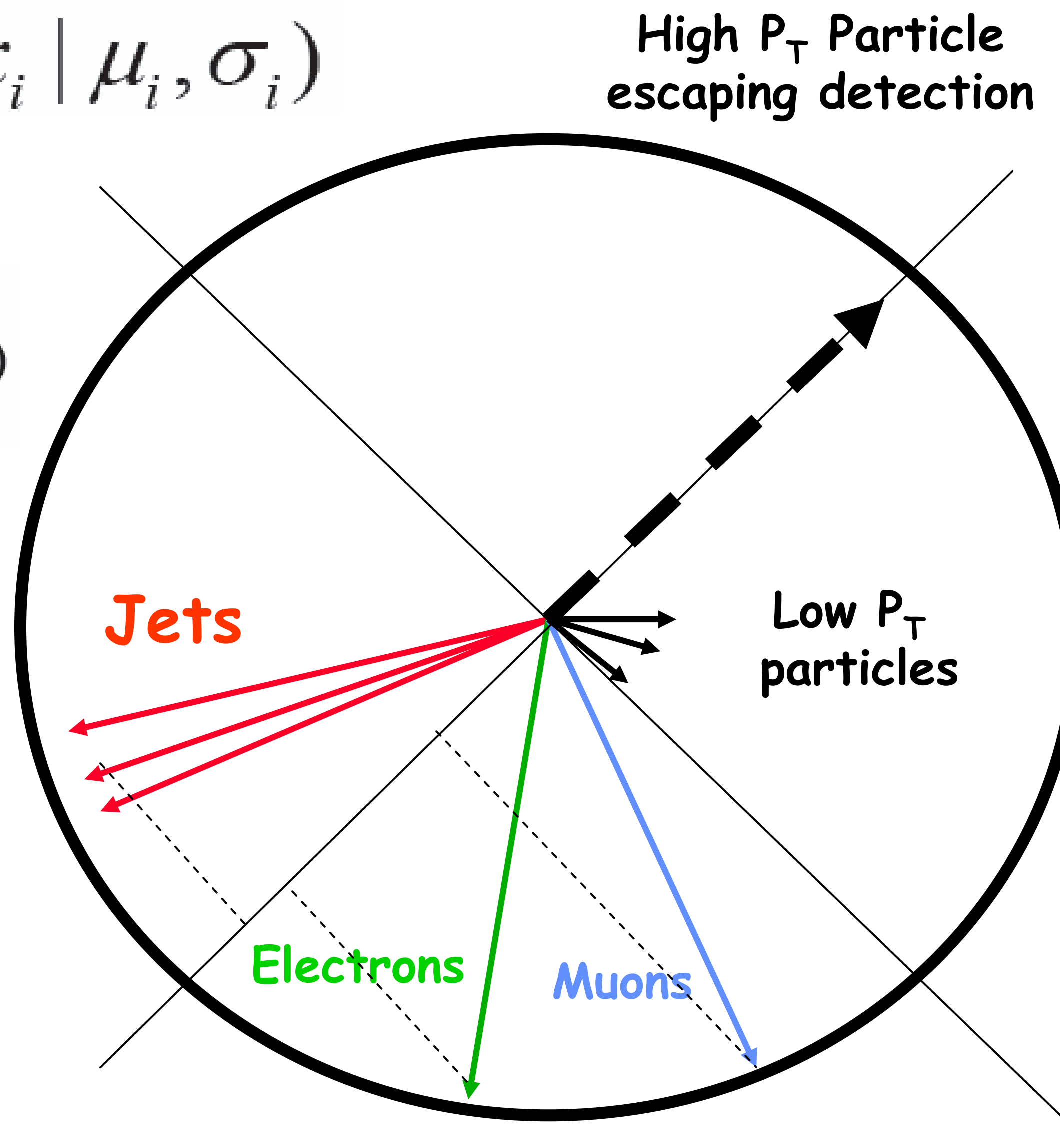
ATLAS Analysis Tutorial, TAU 12/02/07

Introduction

- When we measure the Missing E_T we try make an statement of how much the momentum conservation is broken in the transverse plane due to a particle that escapes detection. However the existence of reconstructed MET does not necessarily imply the presence of a "missing" particle.
- In addition, MET alone does not take into account two very important effects:
  - Two events can have very different amounts of hadronic activity but can have same MET
  - MET does not take into account event topology
- Need to work in terms of a probability that a given final state is inconsistent with a fluctuation of the energy response of objects in the final state: MET Significance
  - Used in CDF and D0 (using objects)

- For illustration purposes, let's assume that the response of each object is expressed in terms of a single Gaussian

$$G(x_i \mid \mu_i, \sigma_i)$$



The MET likelihood distribution becomes

$$L(x) = G(x \mid \mu, \sqrt{\sum_{i=1}^{N} \sigma_i^2 \cos^2(\Delta\phi_i)})$$

$$\mu = \sum_{i=1}^{N} \mu_i \cos(\Delta\phi_i)$$

Where $\Delta\phi_i$ is the angle between each object and MET

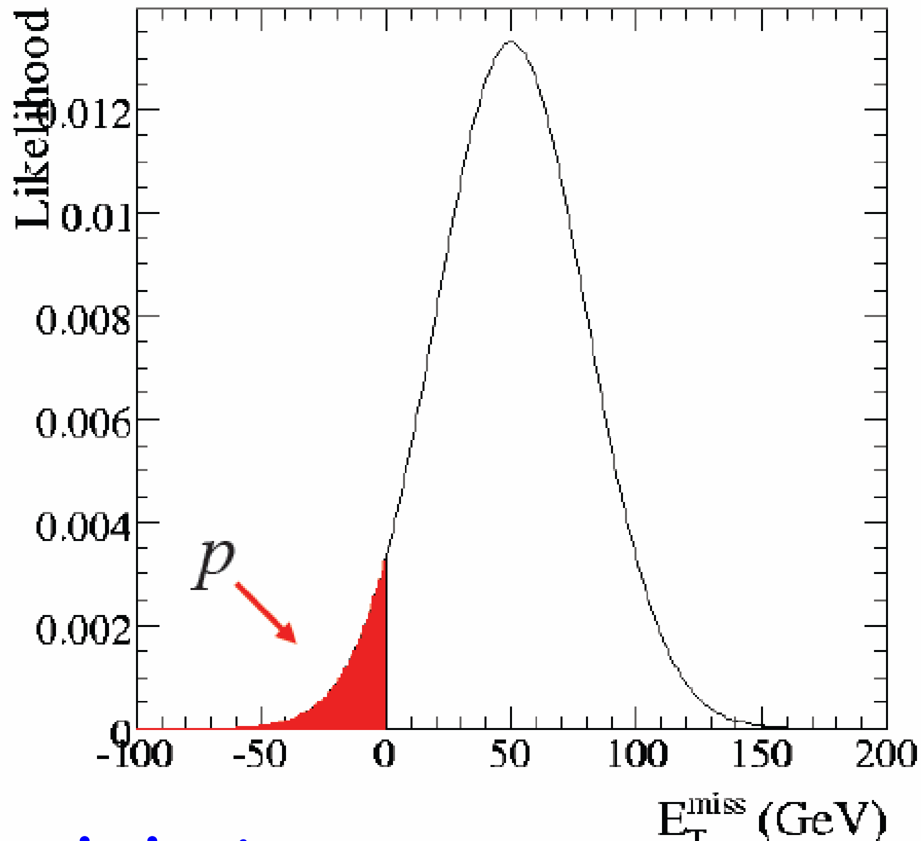
Define

$$\sigma_L = \sqrt{\sum_{i=1}^{N} \sigma_i^2 \cos^2(\Delta\phi_i)}$$

$$\sigma_T = \sqrt{\sum_{i=1}^{N} \sigma_i^2 \sin^2(\Delta\phi_i)}$$

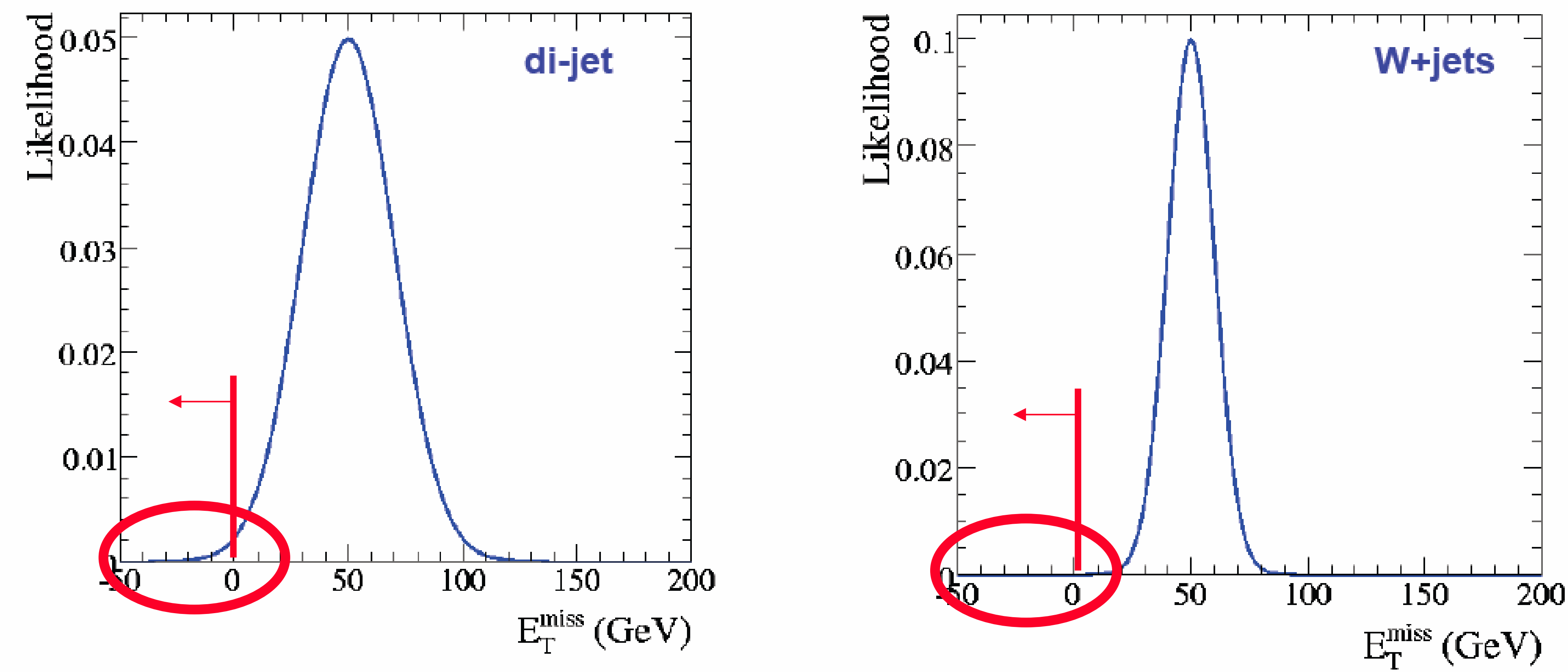
Missing E_T Significance

- The MET hypothesis likelihood can be reconstructed from objects and the p-value can be defined as shown in the figure. MET significance is expressed in terms of number of Gaussian σs (single sided)
  - ➢See X.Chen's talks at JetETmiss meetings for METSig definition and generalization in ATLAS



- D0 uses Log(L(MET)/L(0)) as a discriminator (see A.Schwartzman)
  - ➢This is equivalent to MET significance if particle responses can be parameterized with single Gaussian. This is not optimal for ATLAS (see X.Chen's talks)

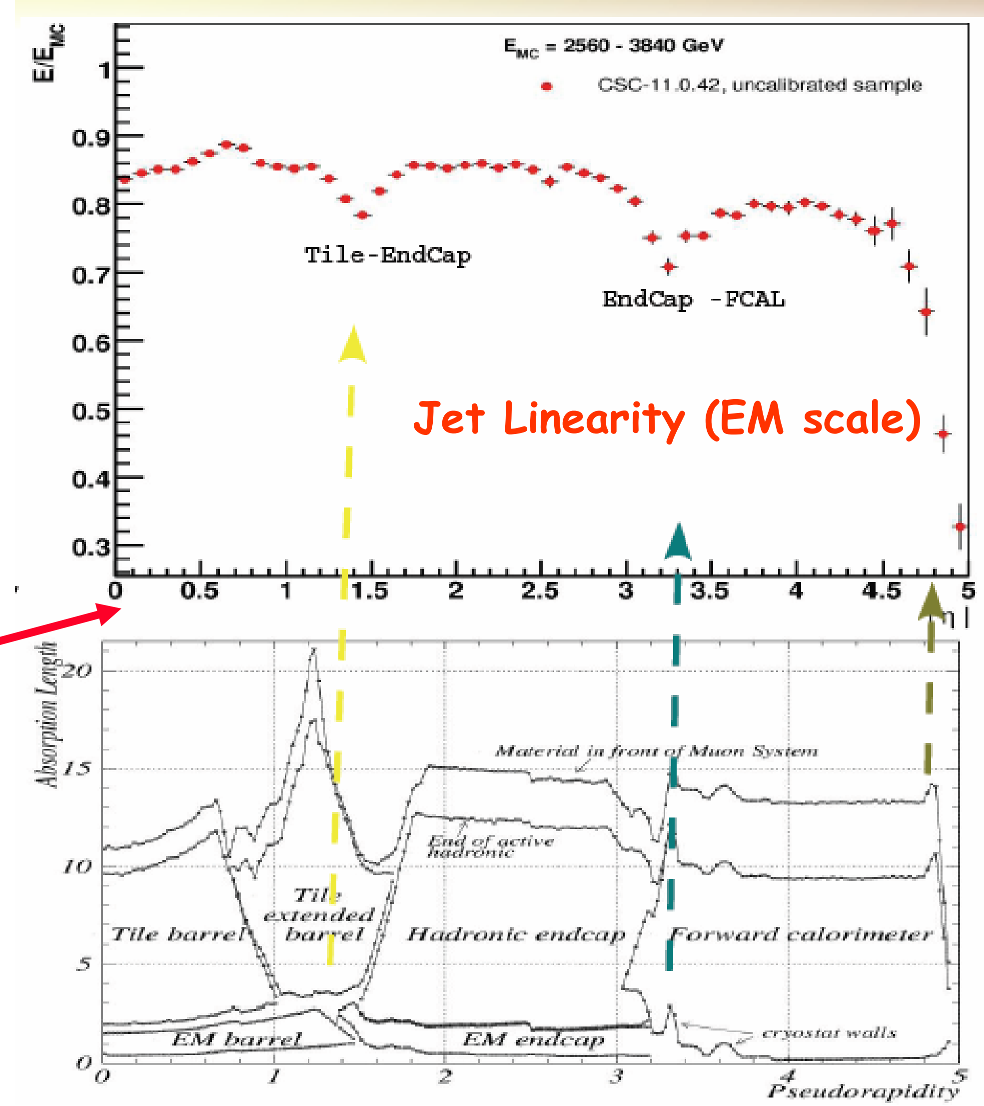
An illustration of different MET significances for two events with same value of reconstructed MET



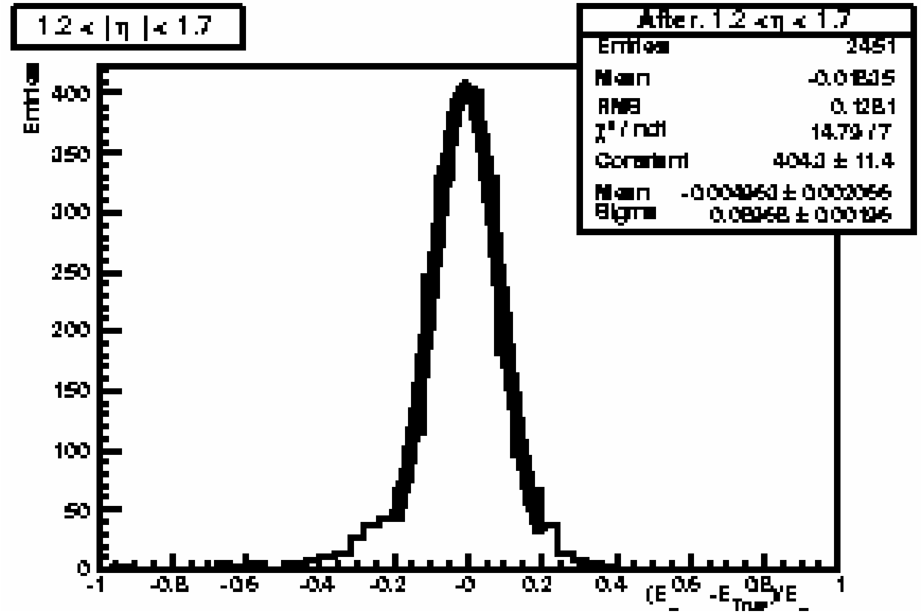
The MET in this di-jet event is more likely to be due to a fluctuation. This event can be rejected by requiring a cut on METSig

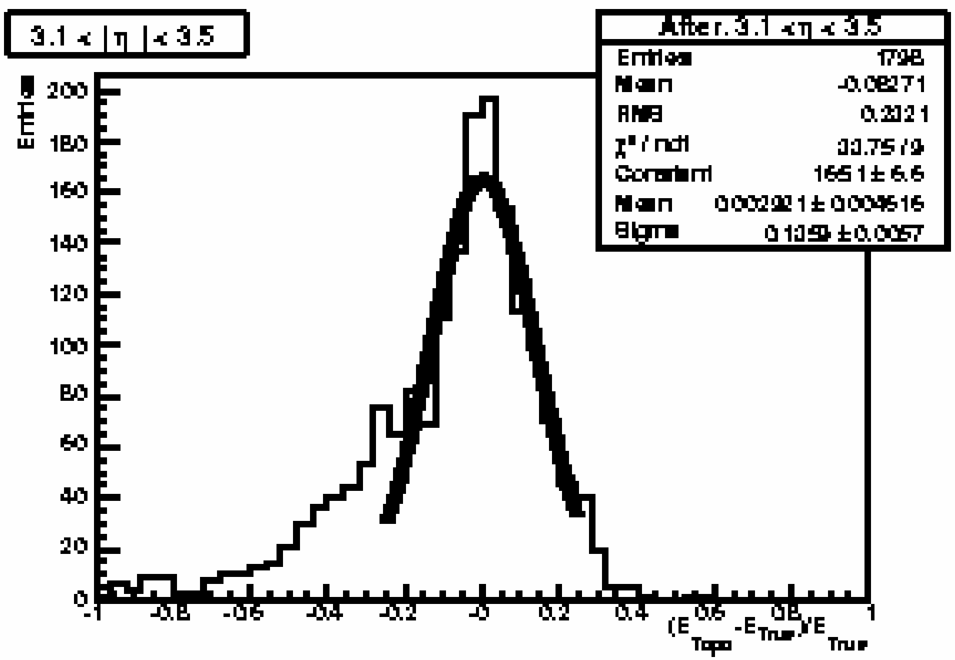
Towards MET Significance

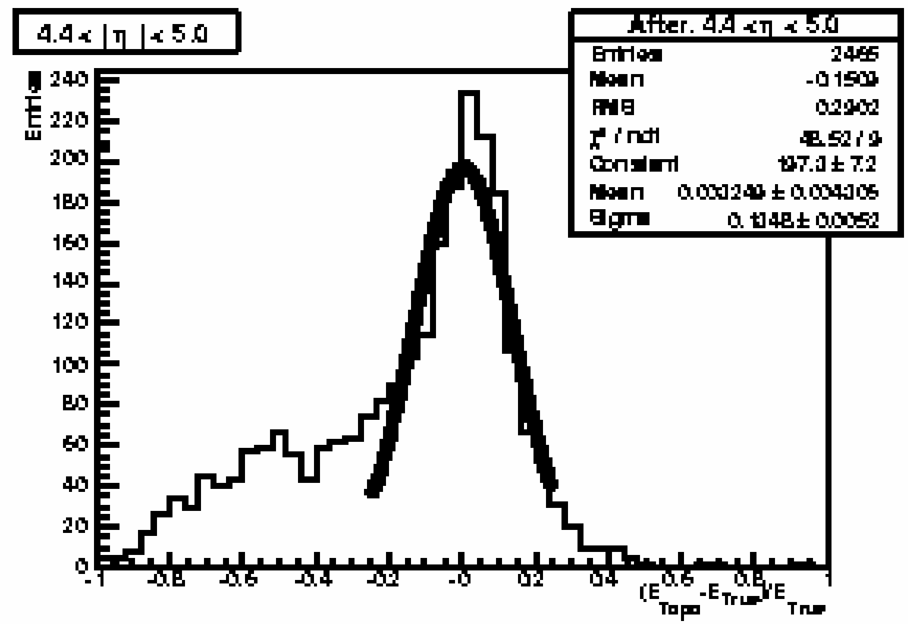
- For definition of MET significance see Ariel's talk
- It is crucial to understand the resolution of different objects as a function of |η| and Energy
- ATLAS has cracks and significant amount of inactive material in front of the calorimeter



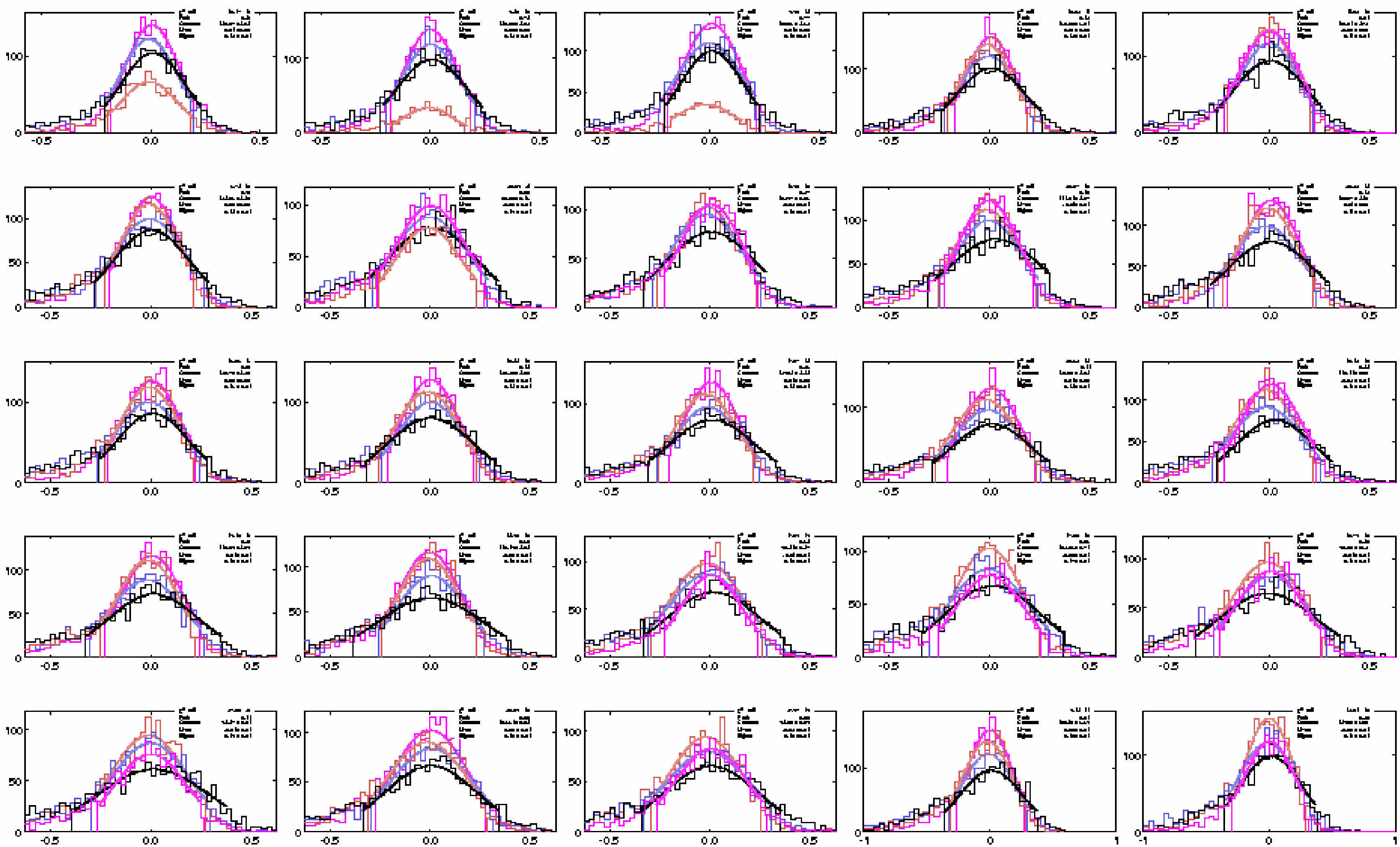
- Shown are fractional deviation from the true energy for single pions with energy 100 GeV (hadronic scale) in "critical" regions



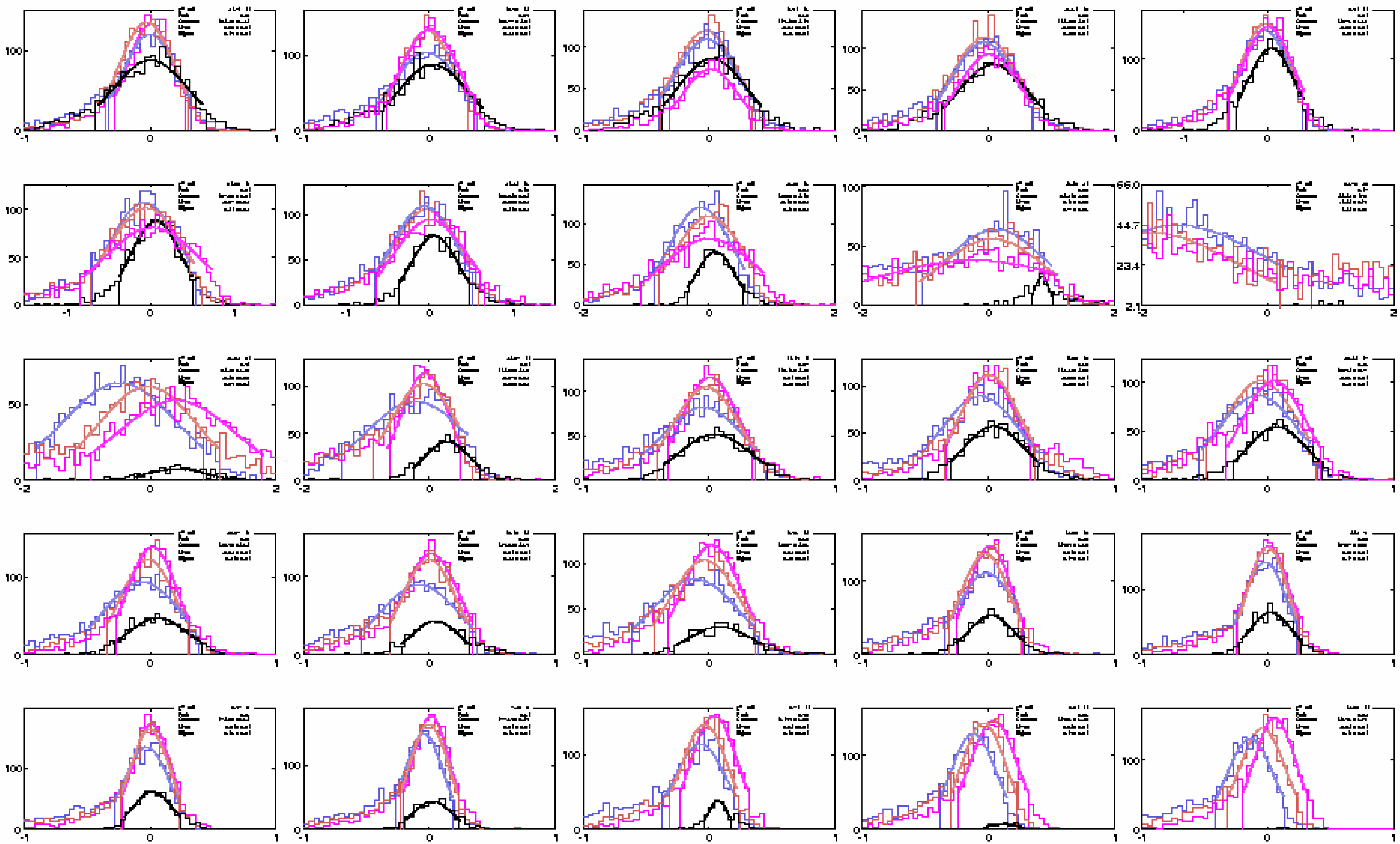




Fractional deviation from the true energy for single electrons with different energies in bins of $0.625<|\eta|<1.25$ in steps of 0.025



Fractional deviation from the true energy for single electrons with different energies in bins of 1.25<|η|<1.875 in steps of 0.025



Resolutions

- Sometimes, resolution function is more complex than a single Gaussian — peak + large tail

- For jets, resolution can be best described by a double Gaussian:

$$G(x_i) = c_i^0 G(x_i \mid \mu_i^0, \sigma_i^0) + c_i^1 G(x_i \mid \mu_i^1, \sigma_i^1)$$

- Consequently, MET likelihood distribution becomes:

$$L(x) = \sum_p c_1^{p(1)} c_2^{p(2)} \cdots c_N^{p(N)} G_p\left(x \mid \sum_{i=1}^{N} \mu_i^{p(i)} \cos(\Delta\phi_i), \sqrt{\sum_{i=0}^{N} \sigma_i^{p(i)^2} \cos^2(\Delta\phi_i)} \right)$$

where p(i) means all the configurations of assigning each of the coefficient terms

- However, more complicated situation is the resolution function is asymmetric (e.g. electrons), but which can be described by a bifurcated Gaussian

Resolutions (cont)

- A bifurcated Gaussian is:

$$L(x) = \begin{cases} \frac{1}{\sqrt{\pi/2}(\sigma_1+\sigma_2)}\exp\left(-\frac{(x-\mu)^2}{2\sigma_1^2}\right), (x<\mu) \\ \frac{1}{\sqrt{\pi/2}(\sigma_1+\sigma_2)}\exp\left(-\frac{(x-\mu)^2}{2\sigma_2^2}\right), (x\geq\mu) \end{cases}$$

- We can exploit the fact that the mean, variance and skew are additive under any convolutions:

$$m = \mu + \sqrt{2/\pi}(\sigma_2-\sigma_1)$$
$$V = \frac{1}{4}(\sigma_1+\sigma_2)^2 + (\frac{3}{4}-\frac{2}{\pi})(\sigma_2-\sigma_1)^2$$
$$\gamma = \sqrt{\frac{2}{\pi}}(\sigma_2-\sigma_1)\left[\frac{1}{4}(\sigma_1+\sigma_2)^2 + (\frac{4}{\pi}-\frac{5}{4})(\sigma_2-\sigma_1)^2\right]$$

- Given m,V and γ, we can solve for μ,σ_1,σ_2. The idea is that we calculate the three cumulants for each object first, add them up, and use a final bifurcated Gaussian as an approximation

Resolution Parameterization

- Electrons are parameterized by a bifurcated Gaussian + Gaussian as a function of (η,E)

$$G(x) = c_1 G(x \mid \mu_1, \sigma_1^L, \sigma_1^R) + c_2 G(x \mid \mu_2, \sigma_2)$$

- Muons, CaloLH muons and Jets are parameterized by a double Gaussian with a common mean as a function of (η, E_T) or (η,E)

- Minijets and unclustered cell energies are parameterized by a single Gaussian as a function of ΣE_T in different detector parts, i.e., barrel, endcap, forward — see talk by B. Mellado

- The various resolution are convoluted systematically for calculation of p-value and significance

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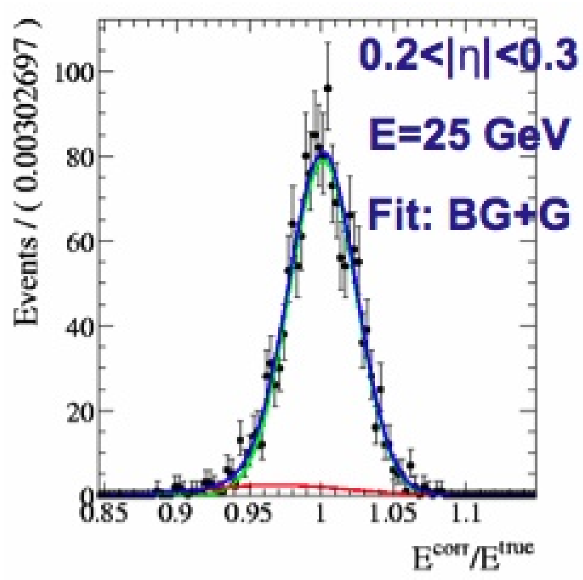
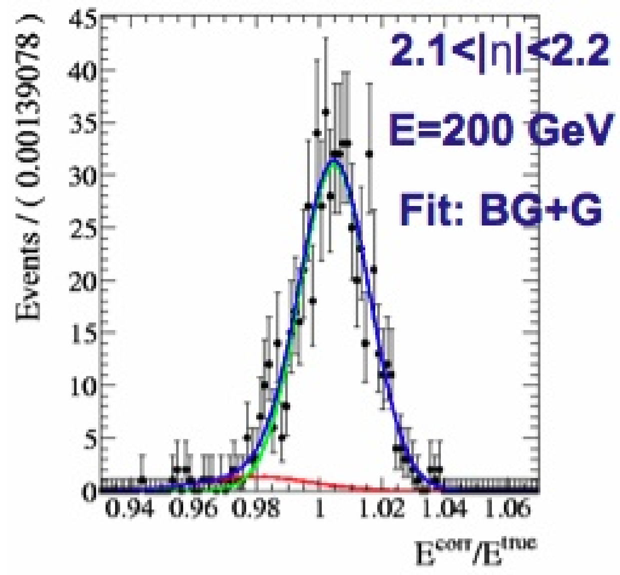
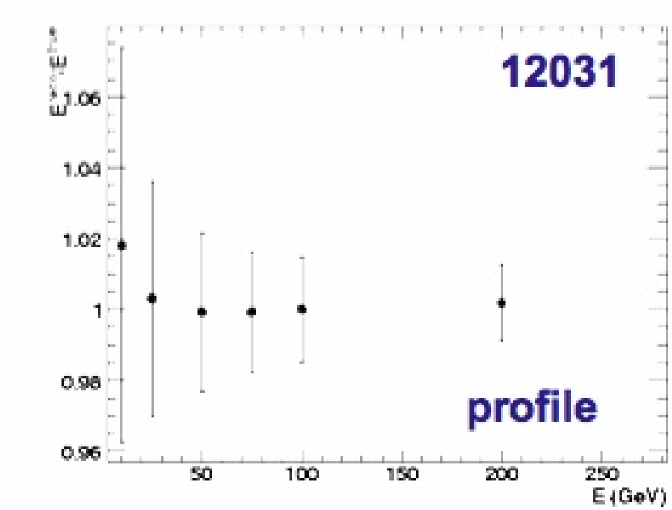
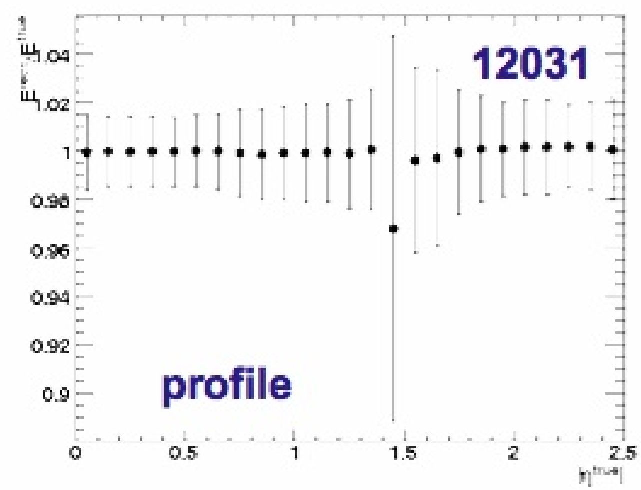
Electron Resolution

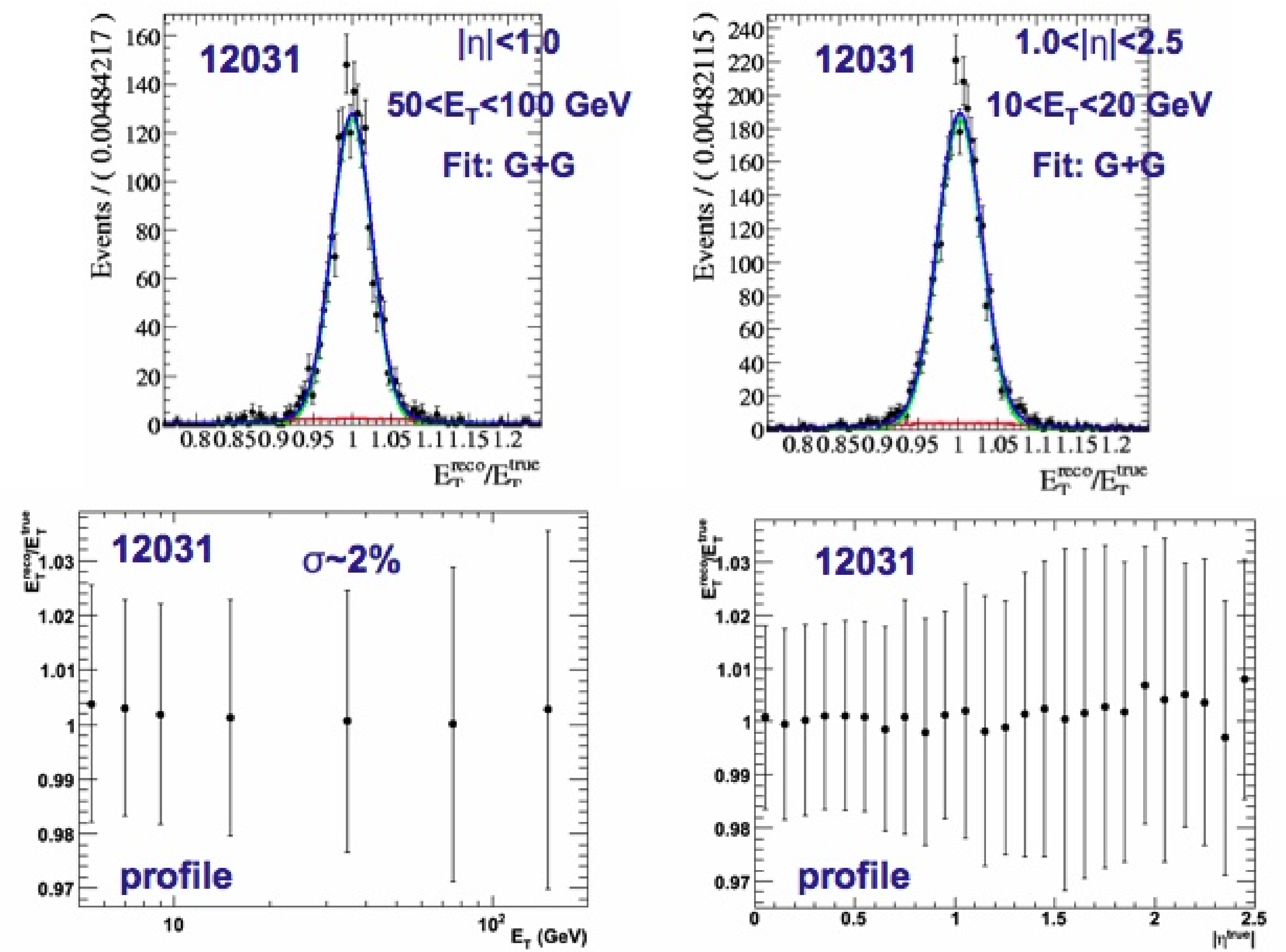




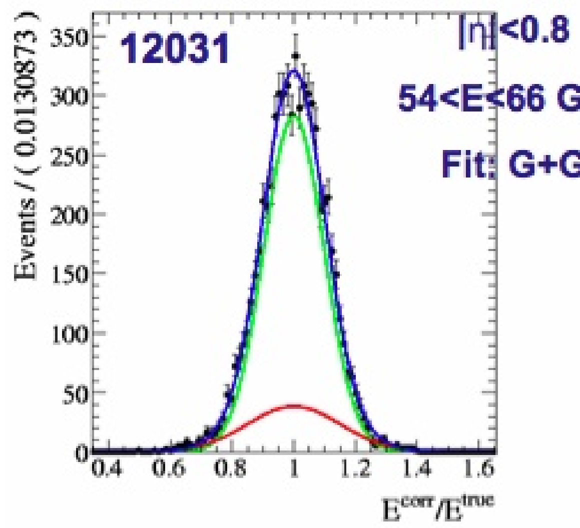


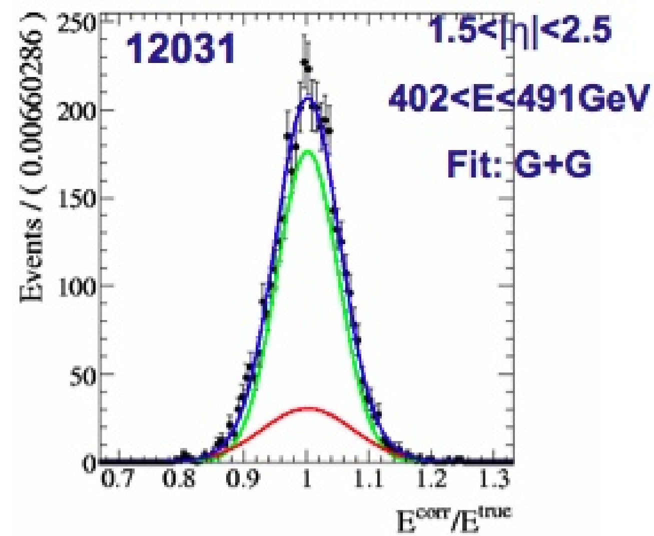


Muon Momentum Resolution

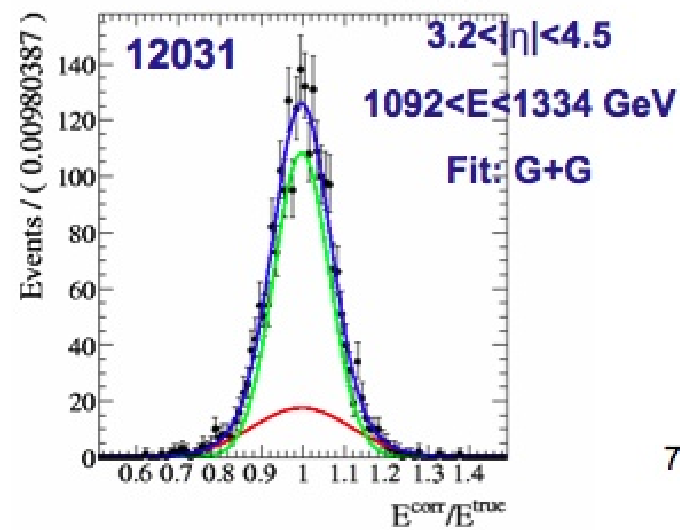


Jet Energy Resolution





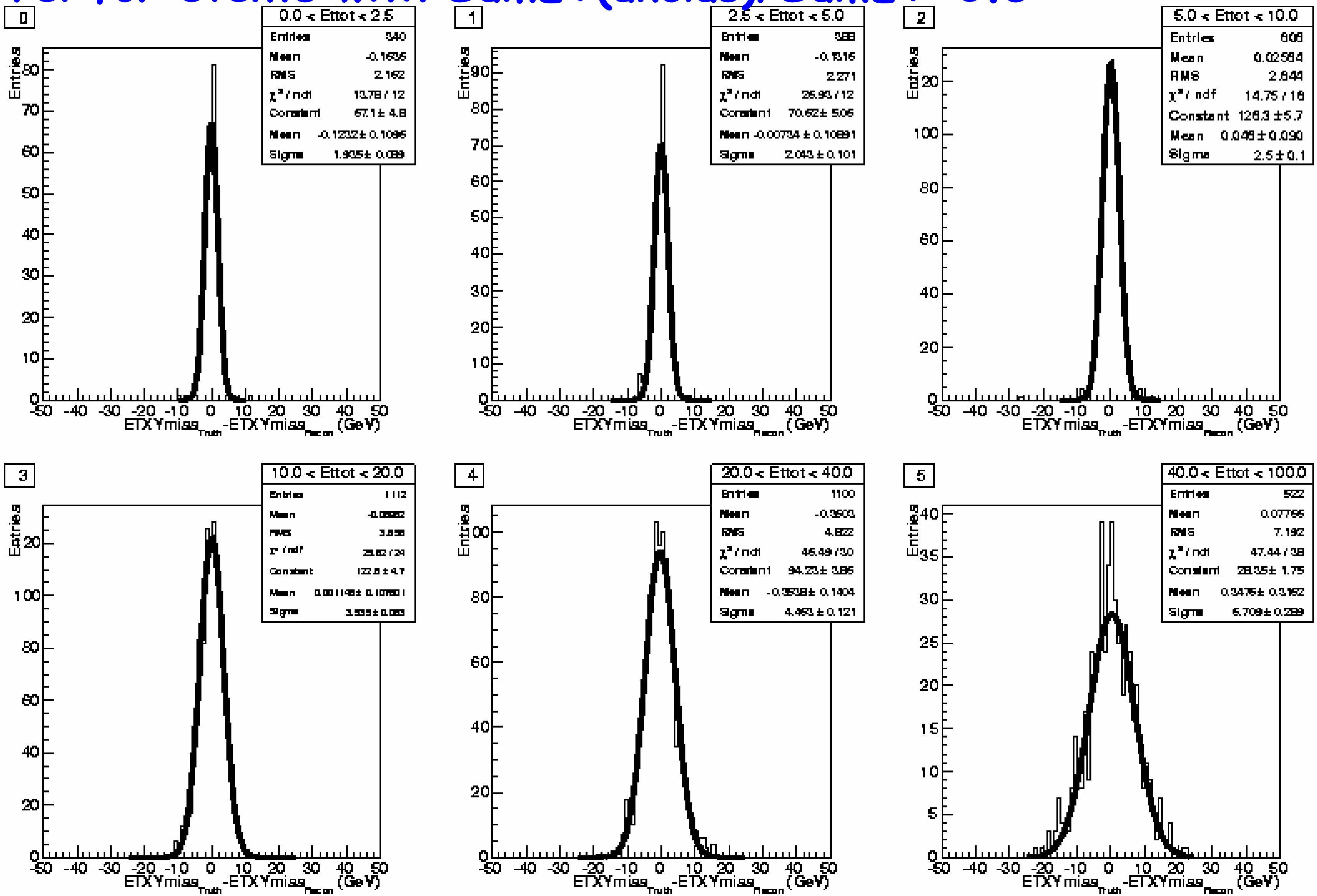
- Jet resolution can be fitted by a double Gaussian in all detector parts
- Double Gaussian is needed for high pt jets. Mostly a single Gaussian is enough for low pt jets



Resolution with Minbias

- Use minimum bias to parameterize the MET resolution of the soft physics component in physics events
  - This is used for the calculation of METSig in ATLAS
  - Divide the contribution from mini-jets and unclustered energy as a function of $|\eta|$
    - Barrel, EndCap and FCAL regions
  - Study the evolution of the MET resolution as a function of each of the components

ObjMET resolution as a function of SumET from mini-jets in the barrel for events with SumET(unclus)/SumET<0.5



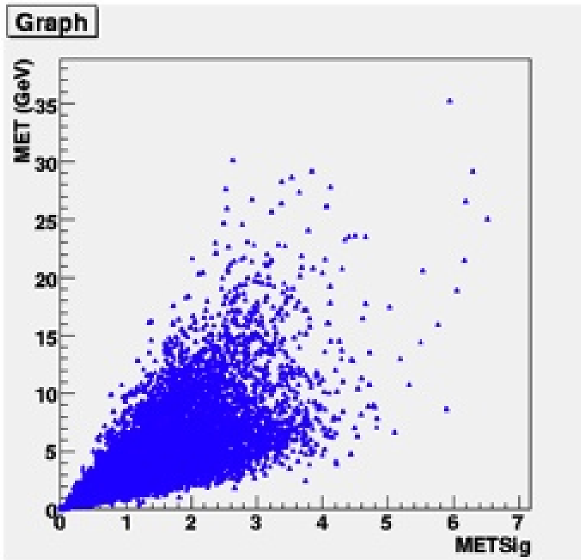
MET Resolution with Minbias

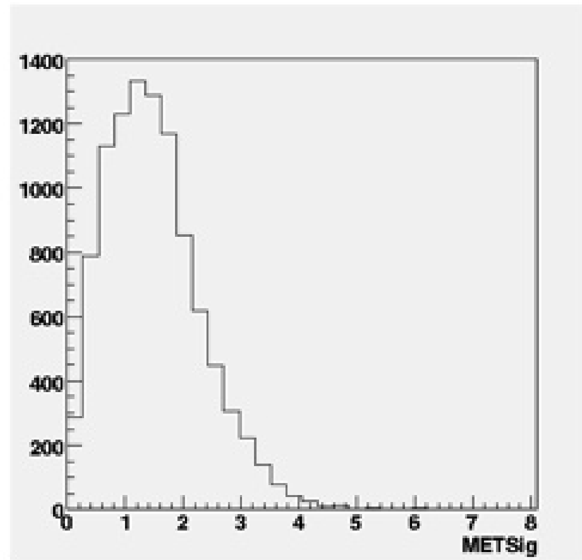
- Obtain $\chi2=42.0$ for simple parameterization $\alpha\sqrt{SumET}$. Otain $\chi2=31.8$ by separating different components of the SumET
- Statistical error on each parameter is about 10%
  - Need more statistics for minimum bias events

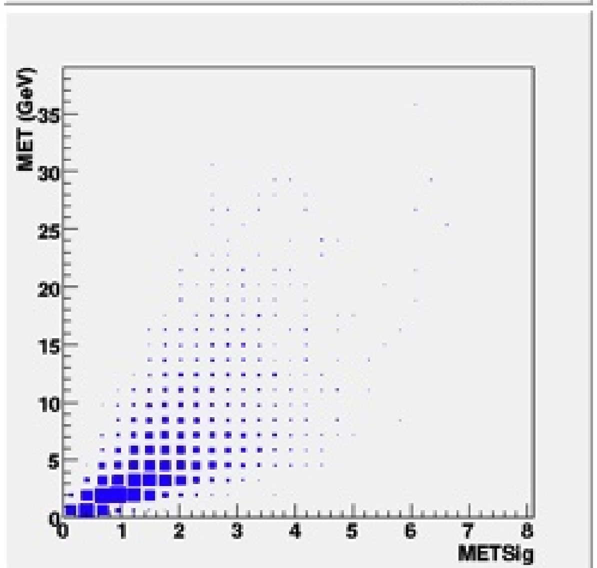
$$\sigma(ETmiss) = 0.6 \oplus 0.48\sqrt{E_{Tsum}^{MjetB}} \oplus 0.45\sqrt{E_{Tsum}^{MjetE}} \oplus 0.58\sqrt{E_{Tsum}^{UncB}} \oplus 0.39\sqrt{E_{Tsum}^{UncE}} \oplus 0.43\sqrt{E_{Tsum}^{UncF}}$$

- Try this for the missing ET significance calculation for the first version of the METSig package
  - May want to try in the future to parametrize the energy response of single pions
    - May be too cumbersome and it is not clear it is necessary

METSig in Minimum Bias

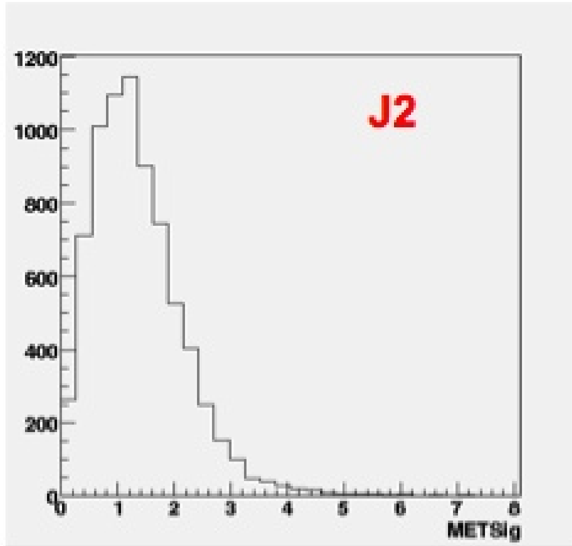


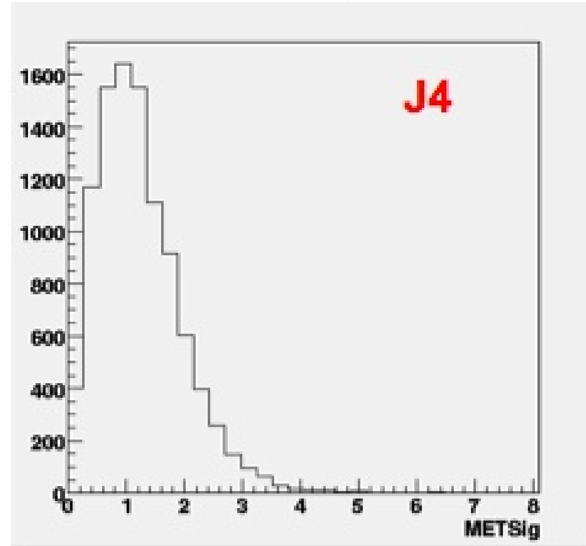


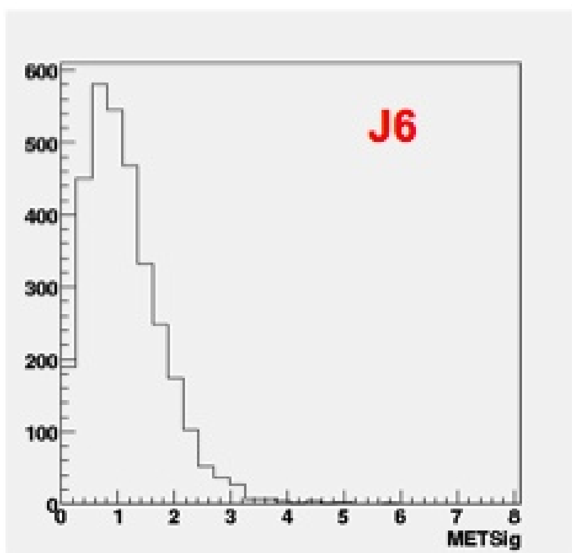


- Peak around 1.2 due to Jacobian "r" — preferentially looking along the direction of final measured MET

METSig for Di-jets

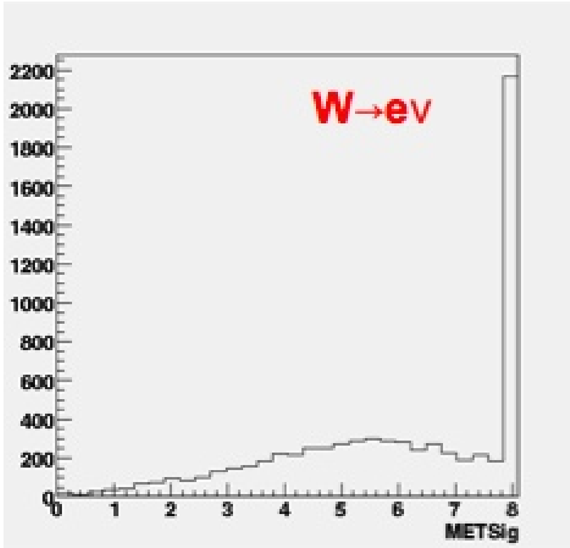


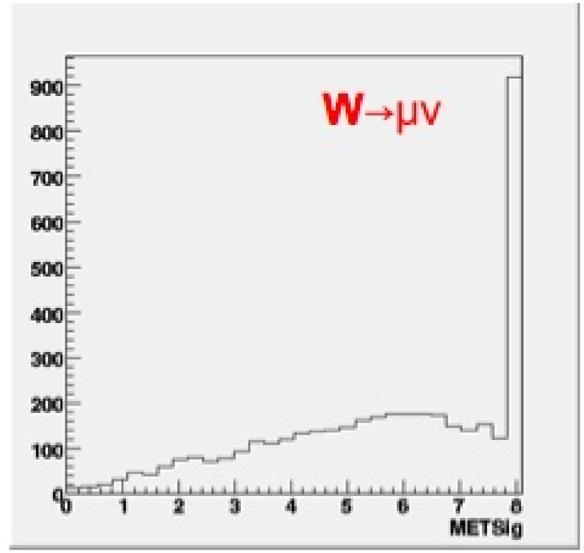


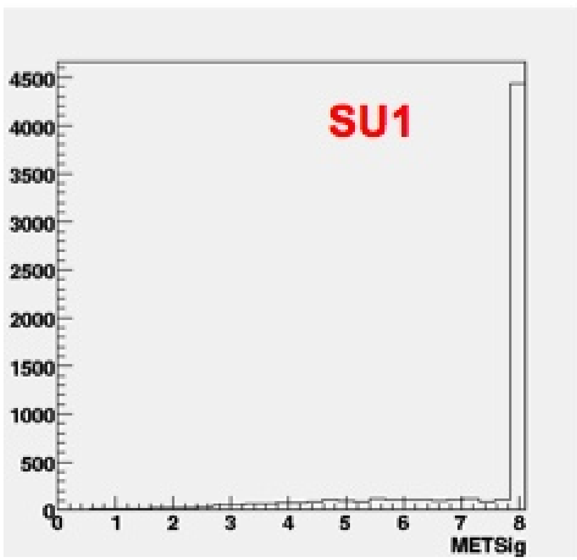


- Similar structure to MB
- The significance does not change very much for higt pt dijets (actually a bit lower than low pt), provided that jets are properly corrected

METSig for W+jets and SUSY (SU1)

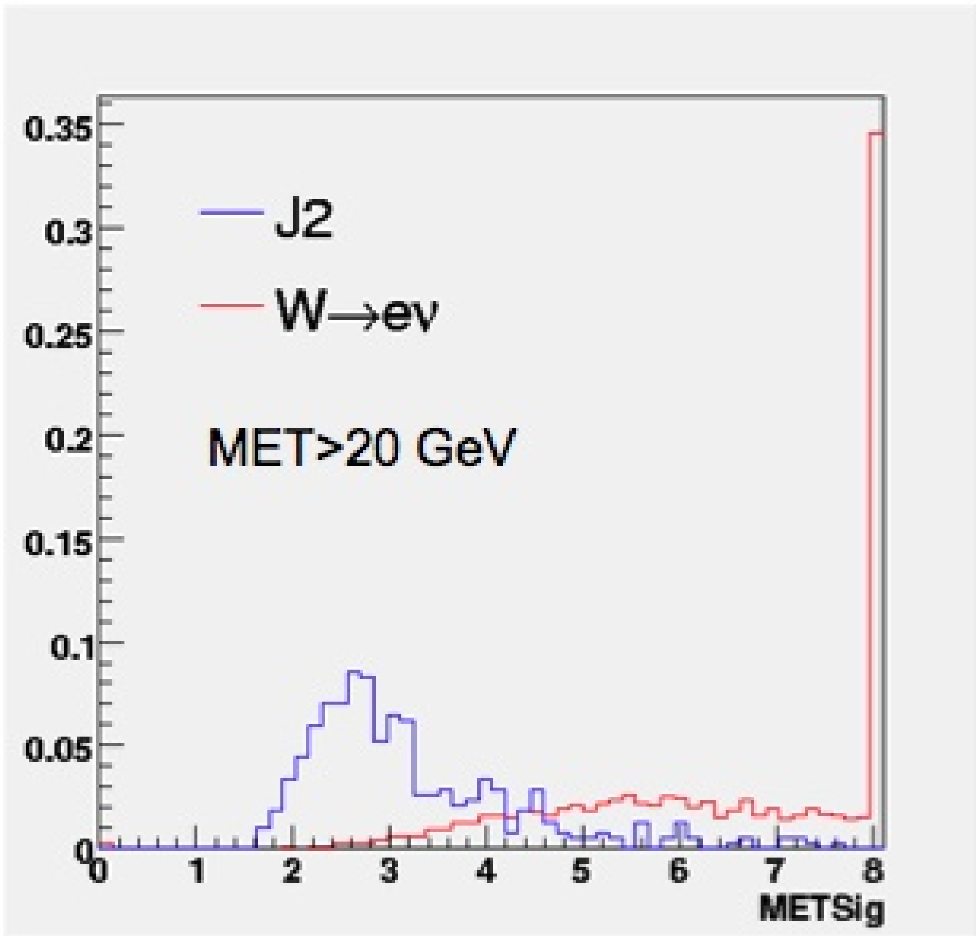


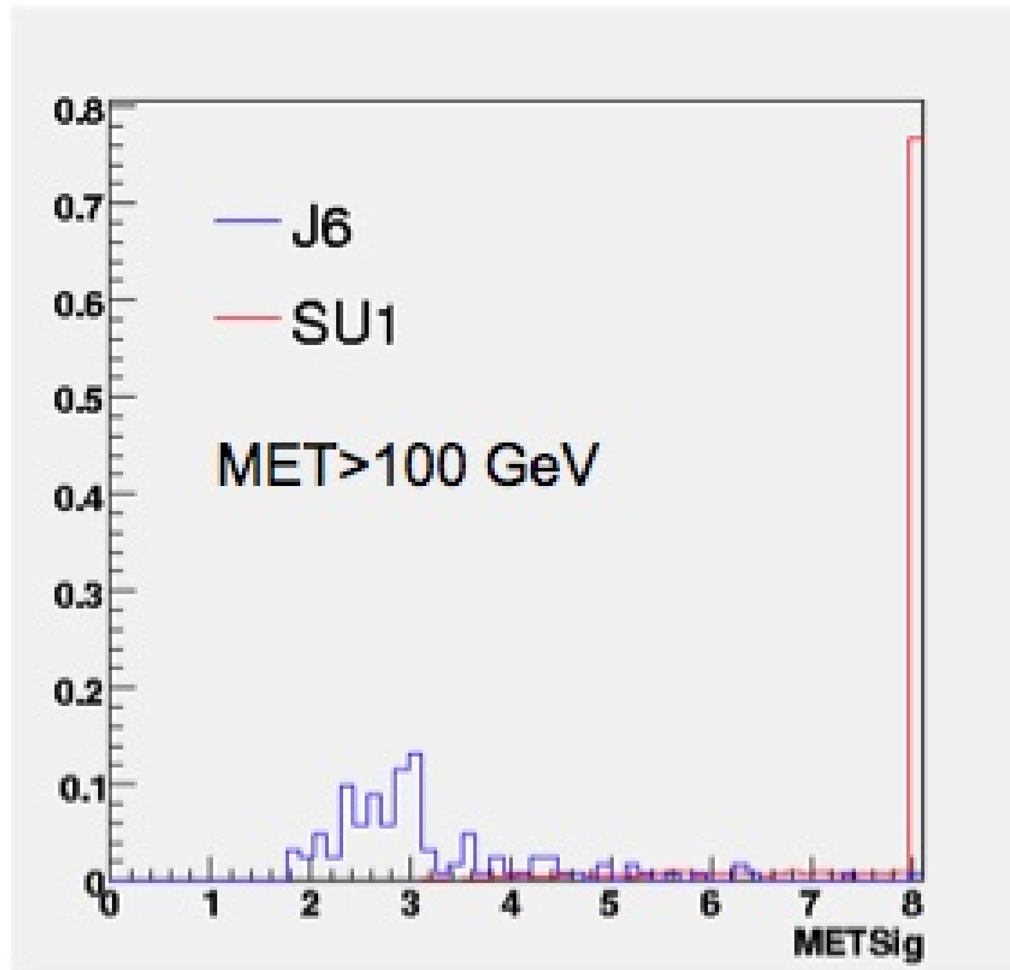




- Cut off at sig=8 due to the sharp rise near 1 of the inverse error function and limitation of machine accuracy. Any thing beyond 8 conveys the info that thi measurement is "very" significant
- A tiny bump near zero indicates a small problem in evaluating $erf^{-1}(x)$ — step accuracy ϵ needs to be tuned for Newton iterations

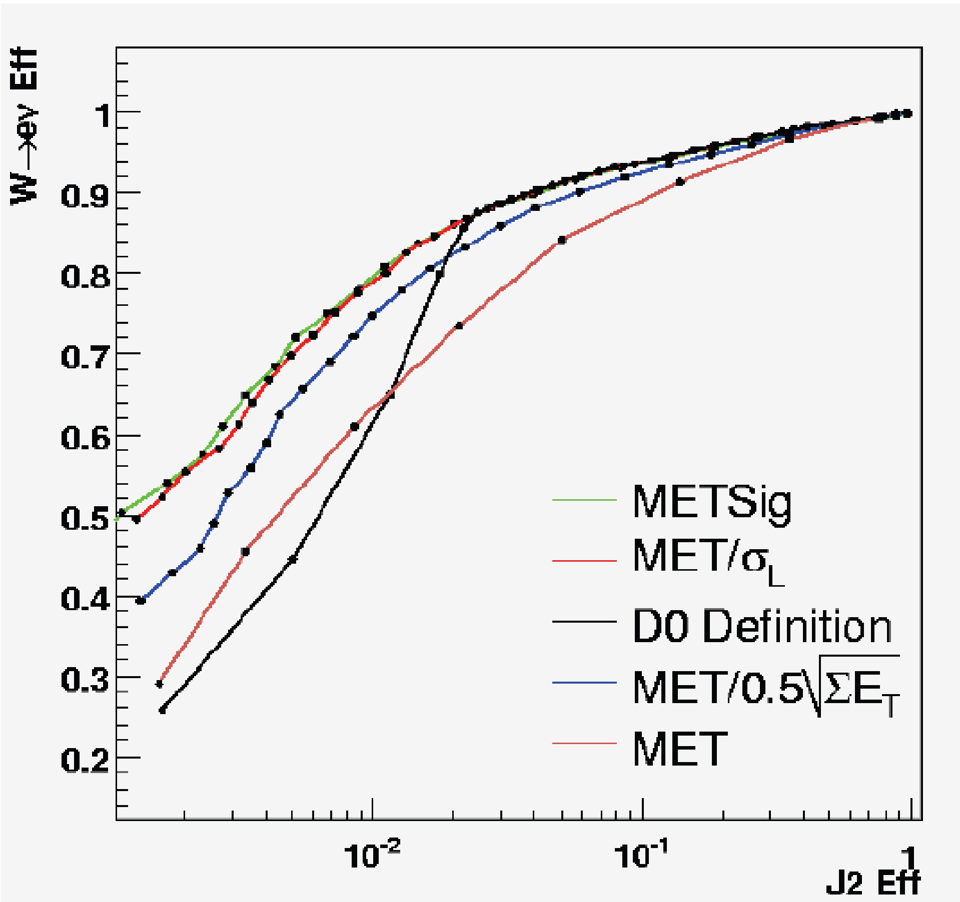
Rejection Power of METSig

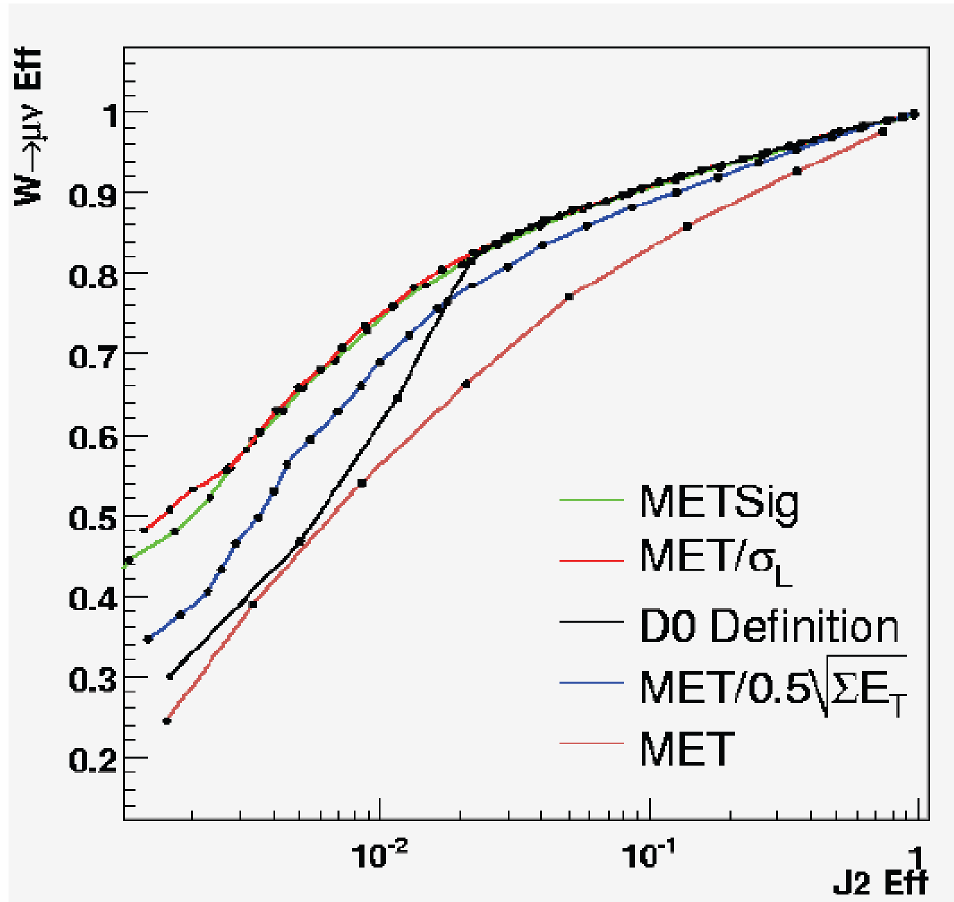




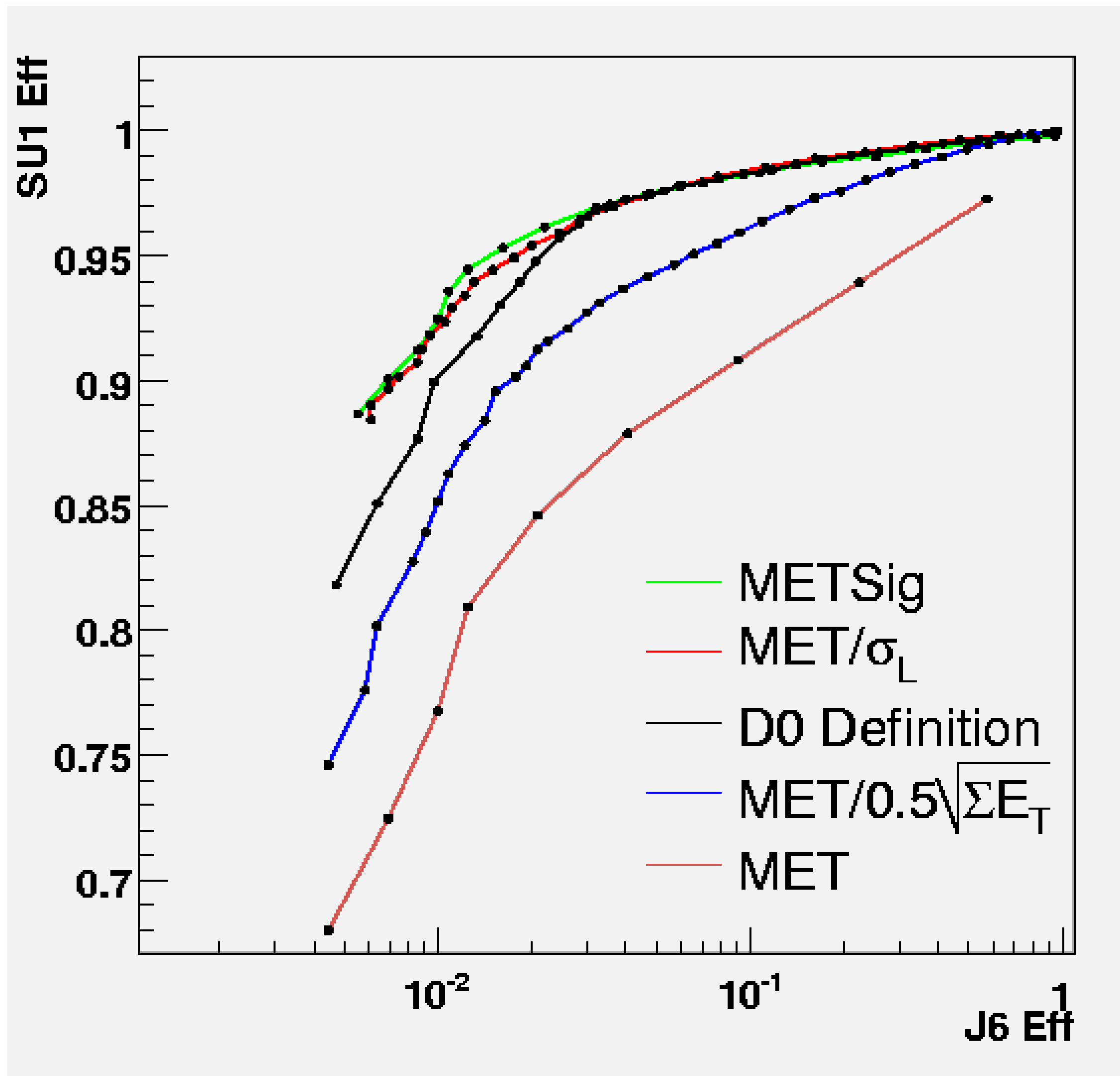
Rejection Power of METSig

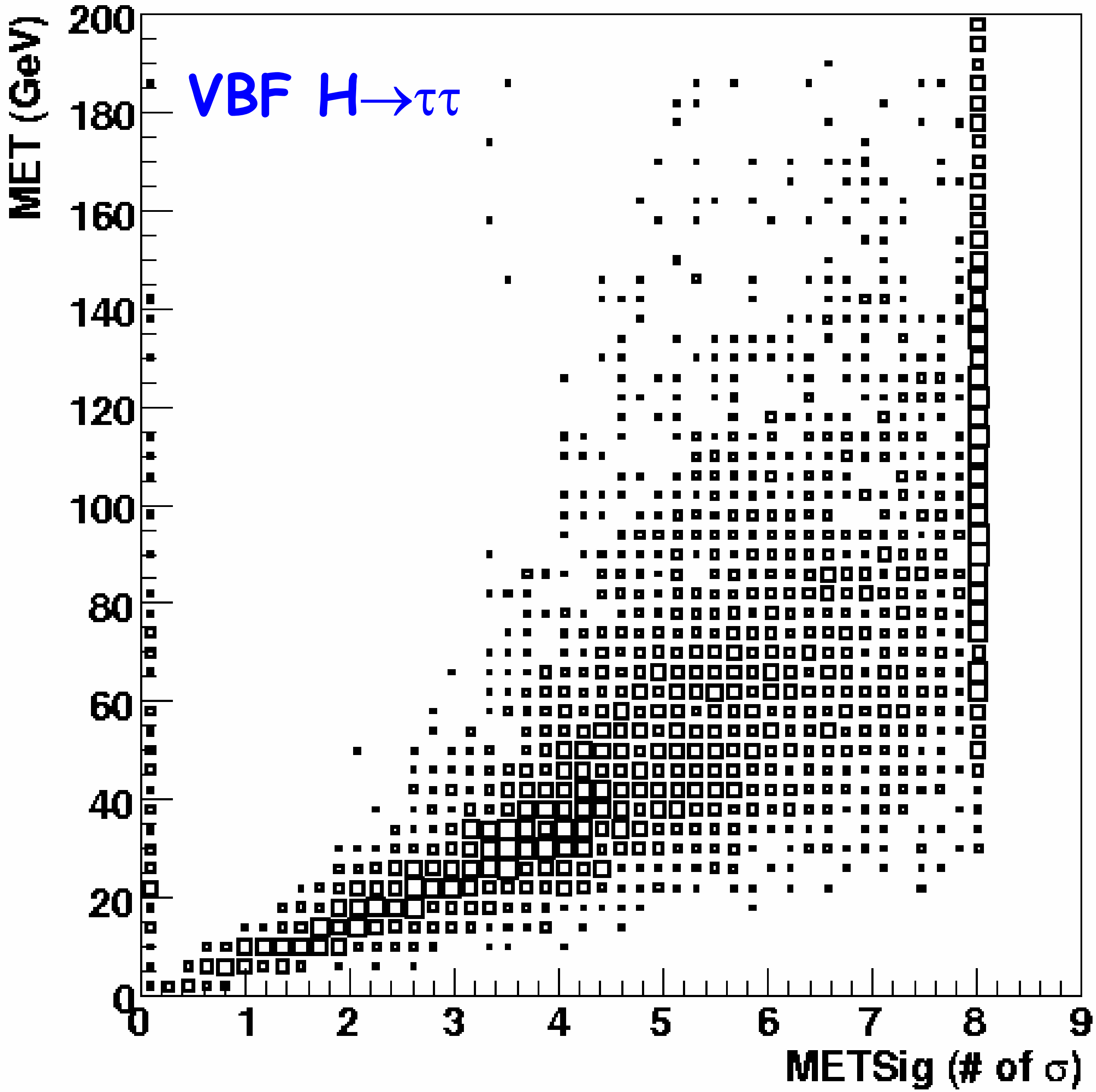
- METSig (or a combination of METsig and MET) is more powerful than an MET cut alone to reject QCD



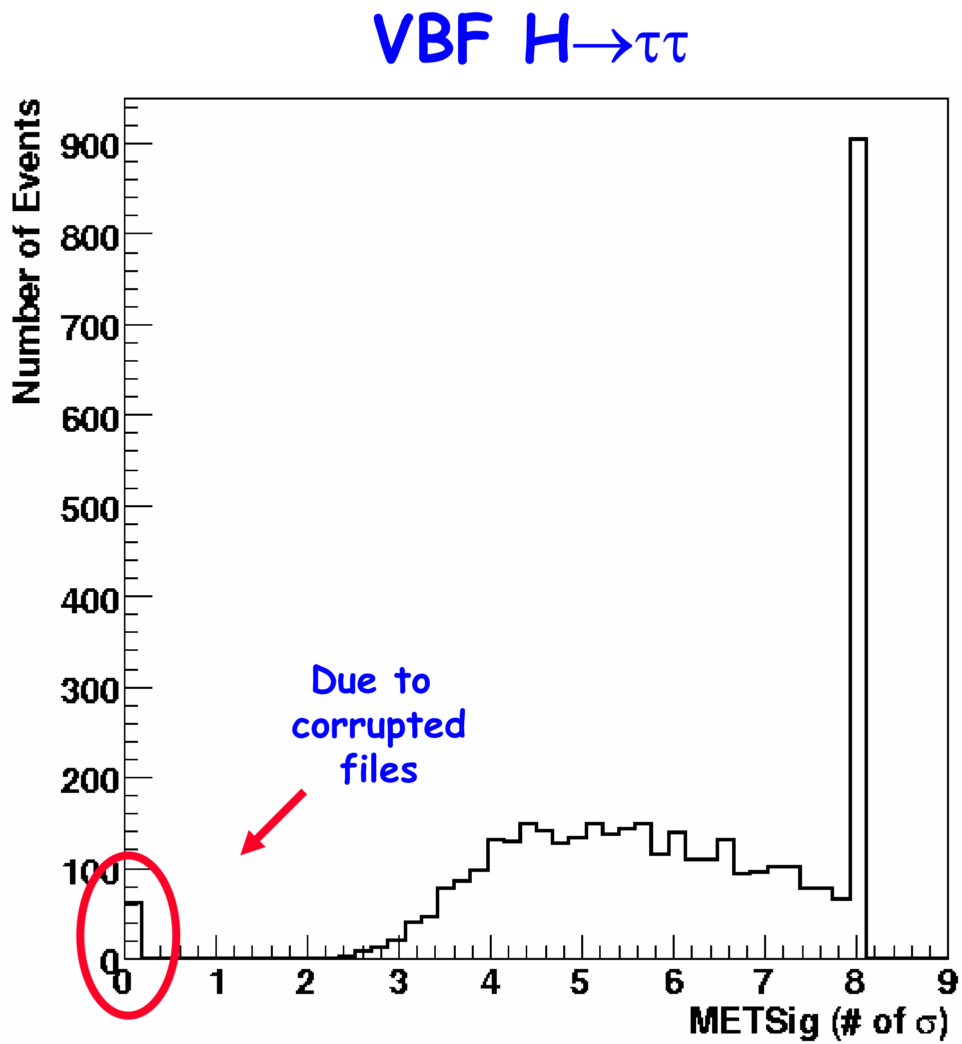


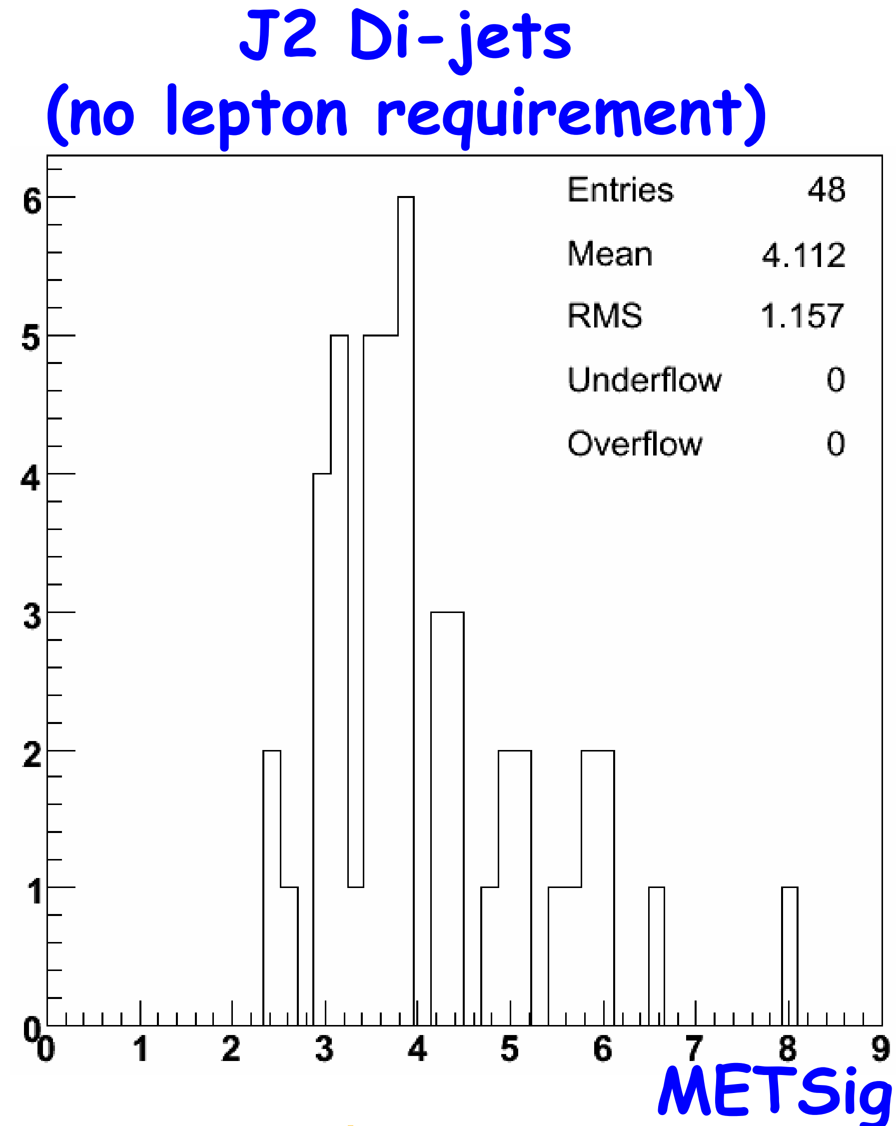
Rejection Power of METSig



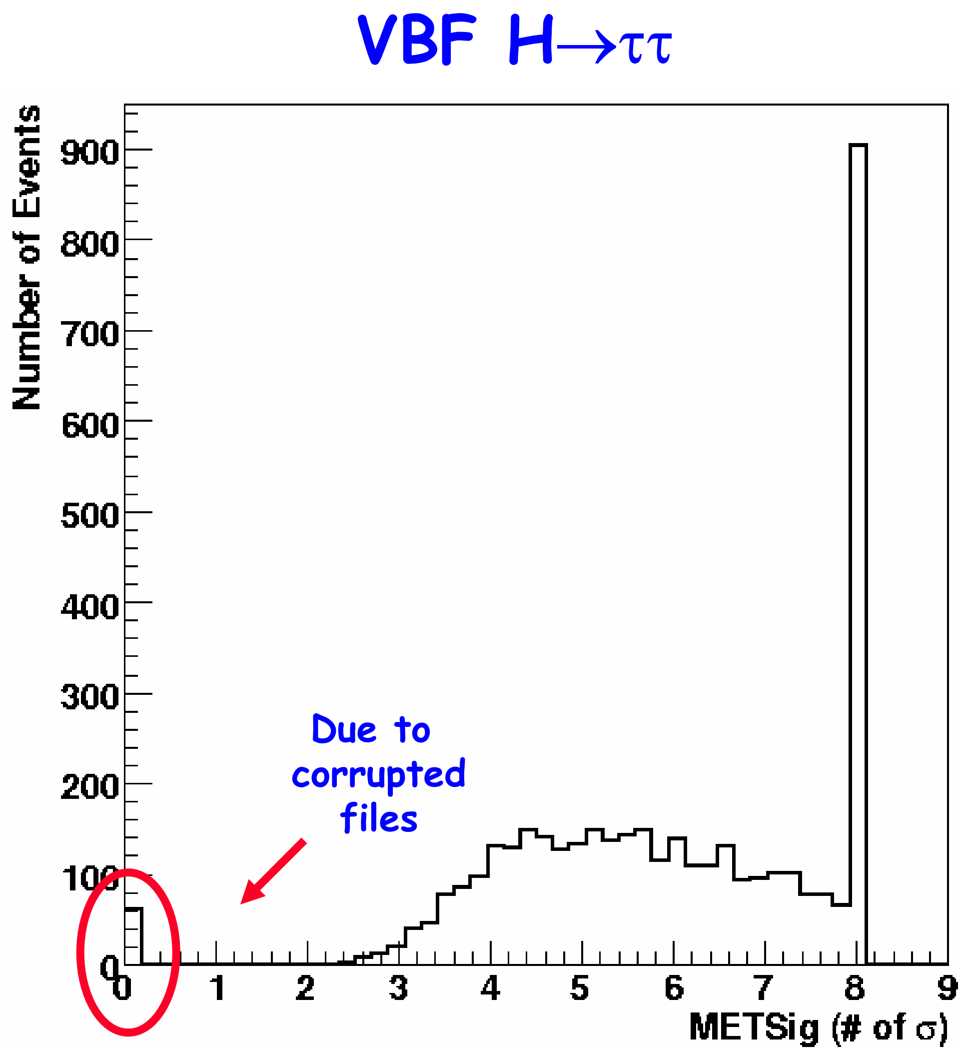

VBF H→ττ
MET (GeV)
METSig (# of σ)
0
20
40
60
80
100
120
140
160
180
200
0
1
2
3
4
5
6
7
8
9

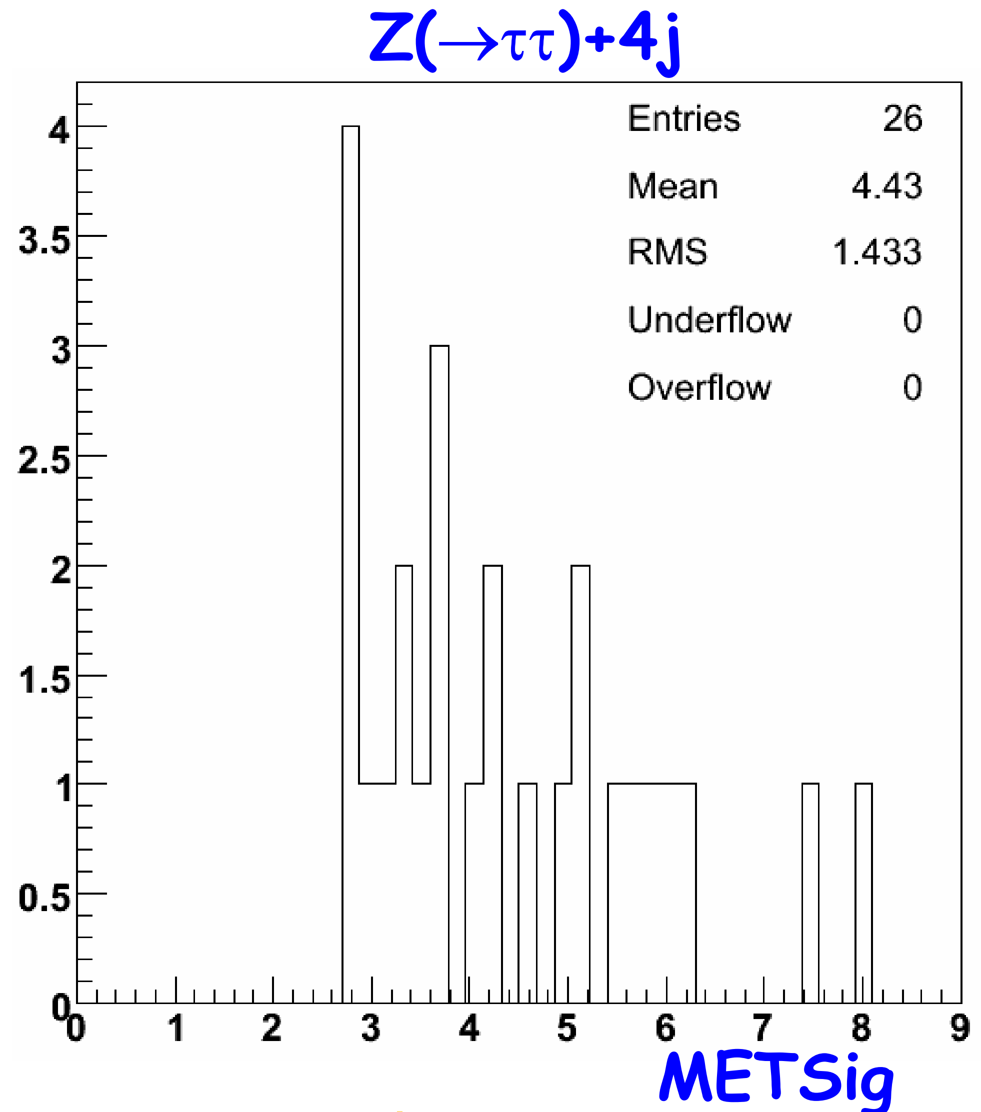
METSig with MET>30 GeV



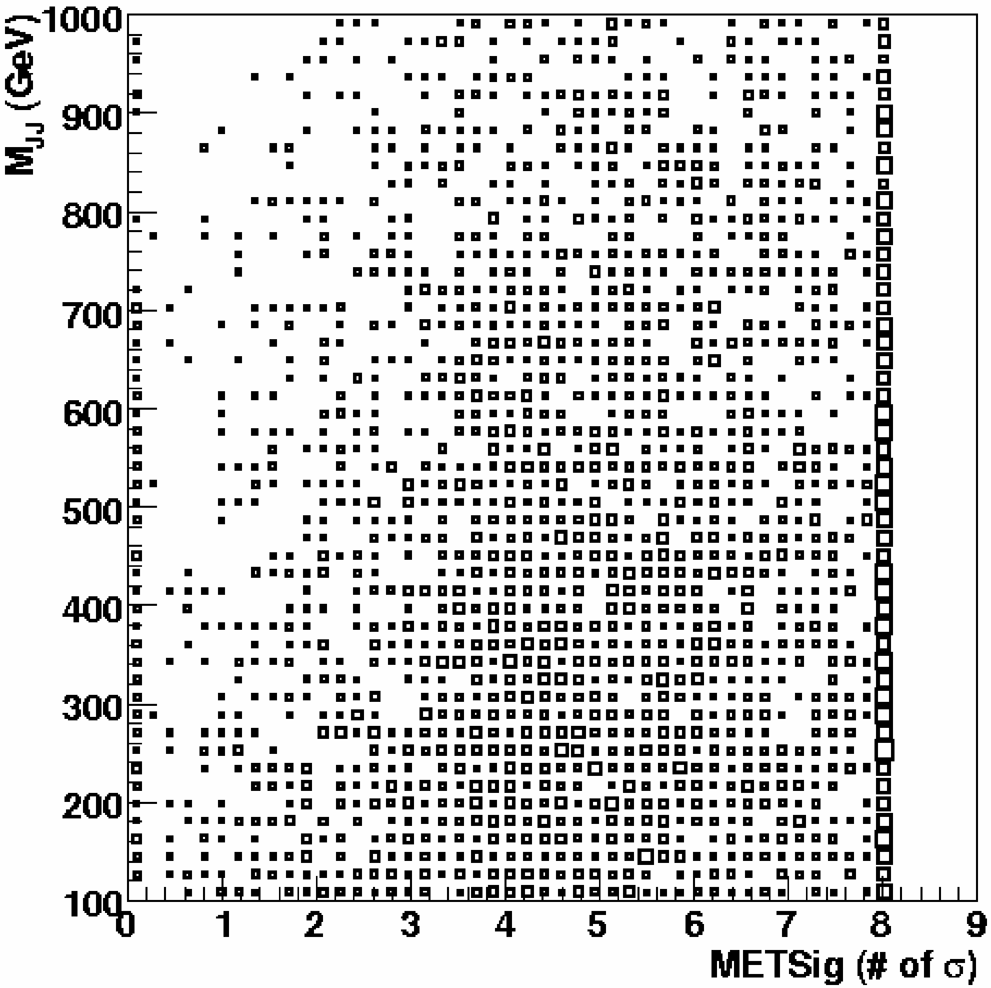


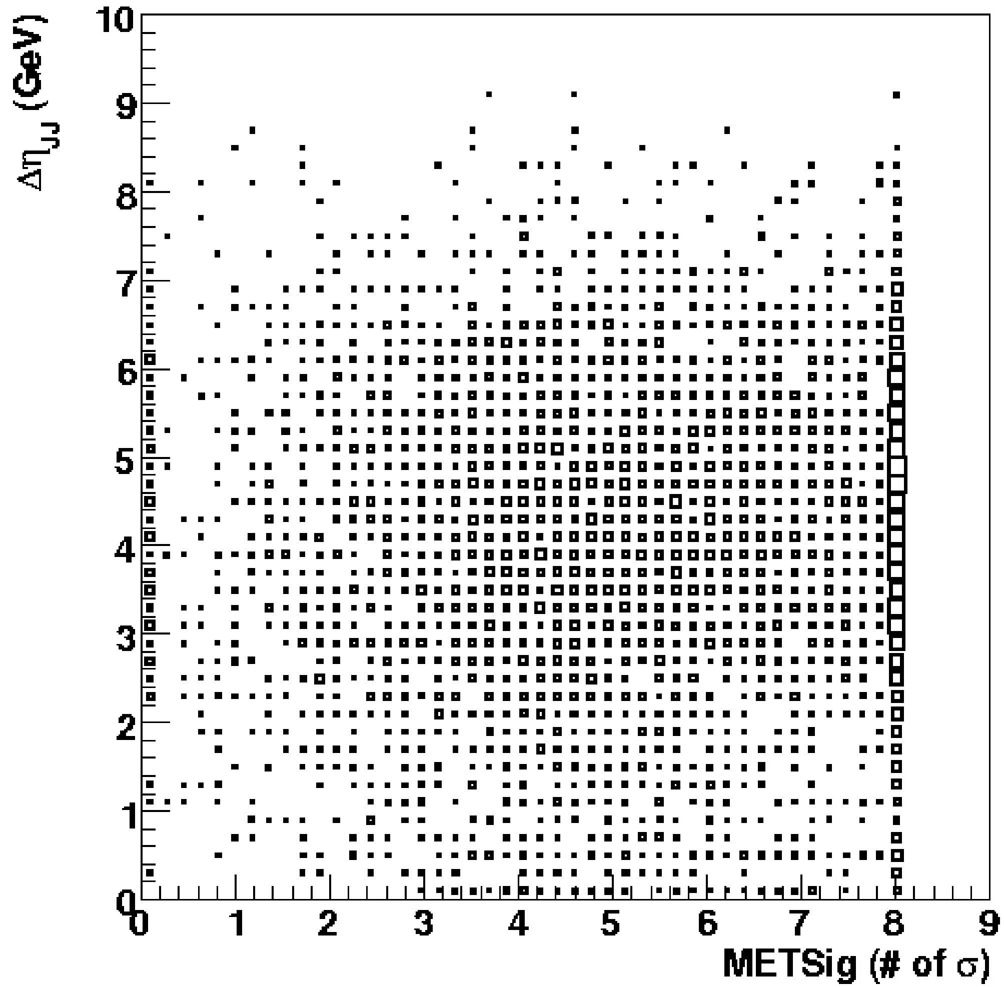
METSig with MET>30 GeV

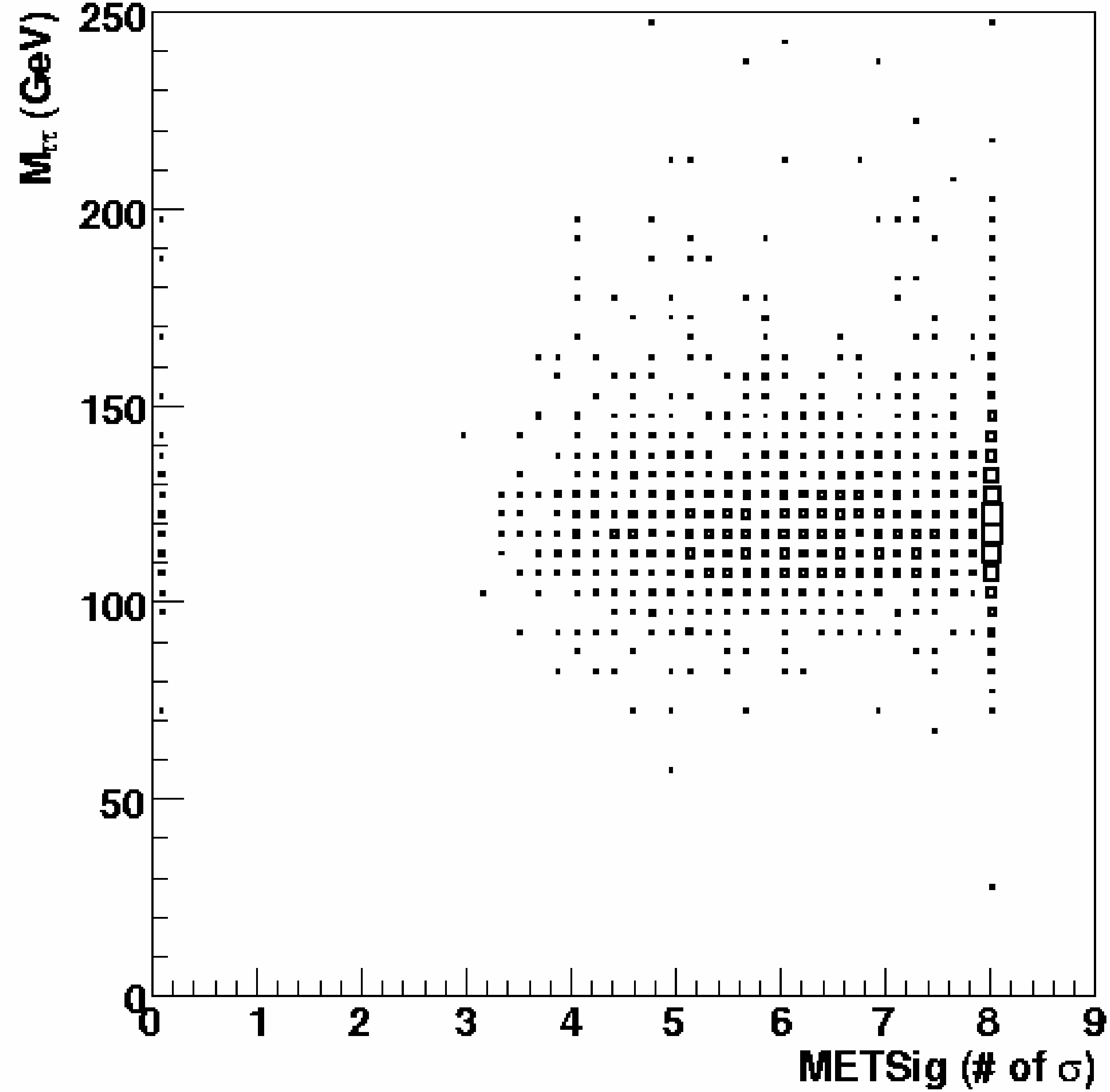




- Do not observe obvious correlation between METSig and the leading discriminating variables







$M_{\tau\tau}$ (GeV)
250
200
150
100
50
0
0 1 2 3 4 5 6 7 8 9
METSig (# of σ)

- For illustration purposes, let's assume that the response of each object is expressed in terms of a single Gaussian

$$G(x_i \mid \mu_i, \sigma_i)$$

The MET likelihood distribution becomes

$$L(x) = G\left(x \mid \mu, \sqrt{\sum_{i=1}^{N} \sigma_i^2 \cos^2(\Delta\phi_i)}\right)$$

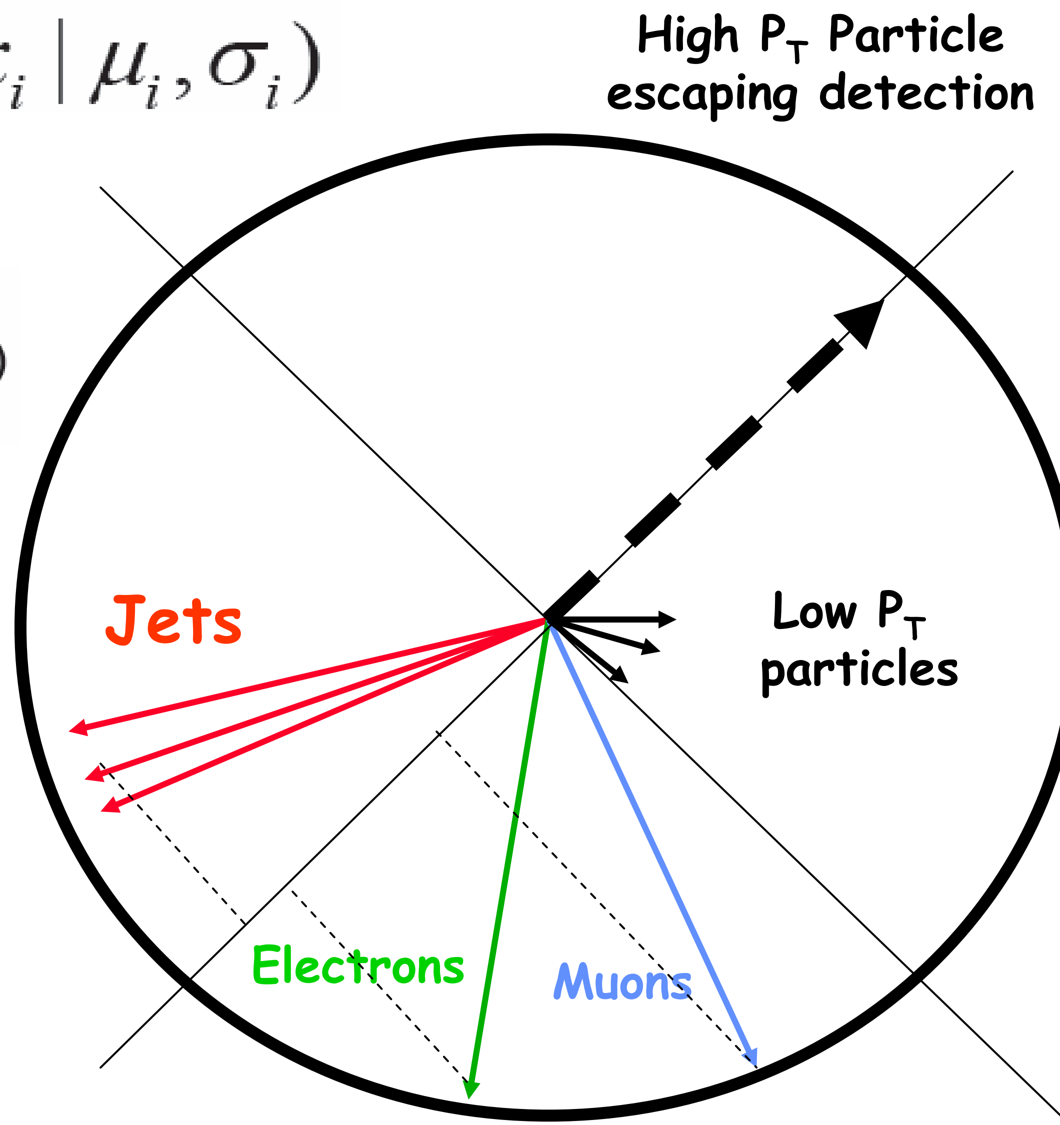
$$\mu = \sum_{i=1}^{N} \mu_i \cos(\Delta\phi_i)$$

Where $\Delta\phi_i$ is the angle between each object and MET

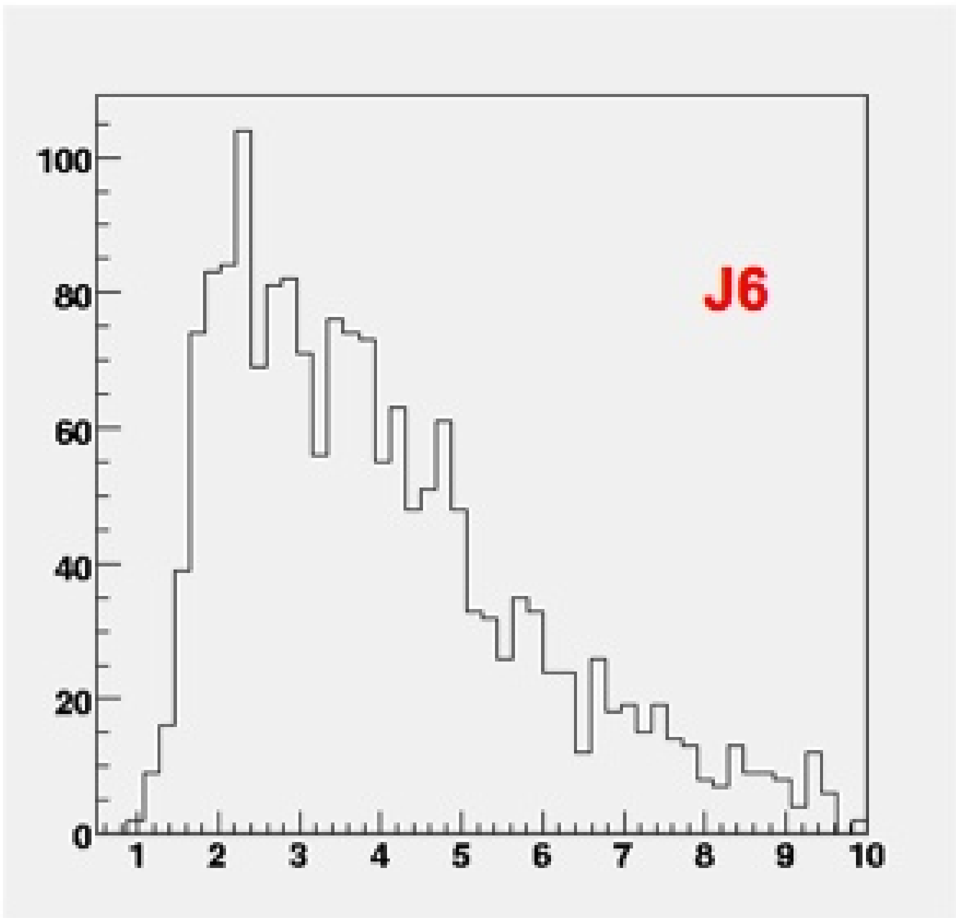
Define

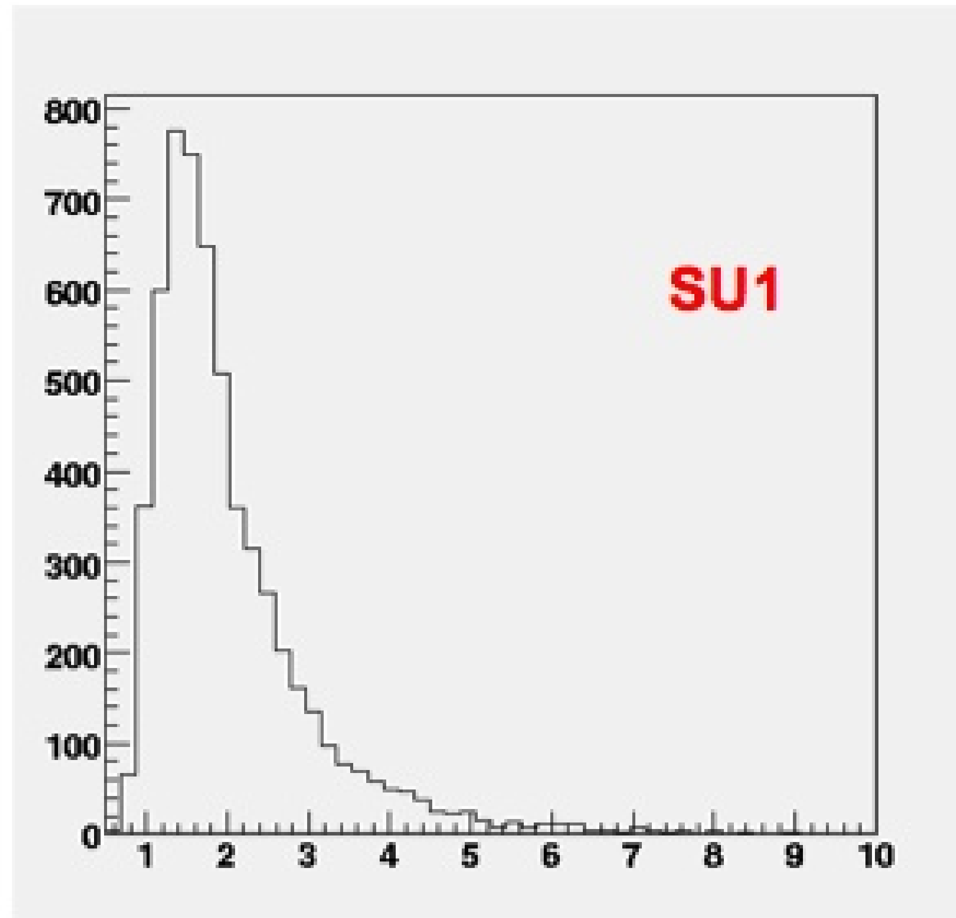
$$\sigma_L = \sqrt{\sum_{i=1}^{N} \sigma_i^2 \cos^2(\Delta\phi_i)}$$

$$\sigma_T = \sqrt{\sum_{i=1}^{N} \sigma_i^2 \sin^2(\Delta\phi_i)}$$

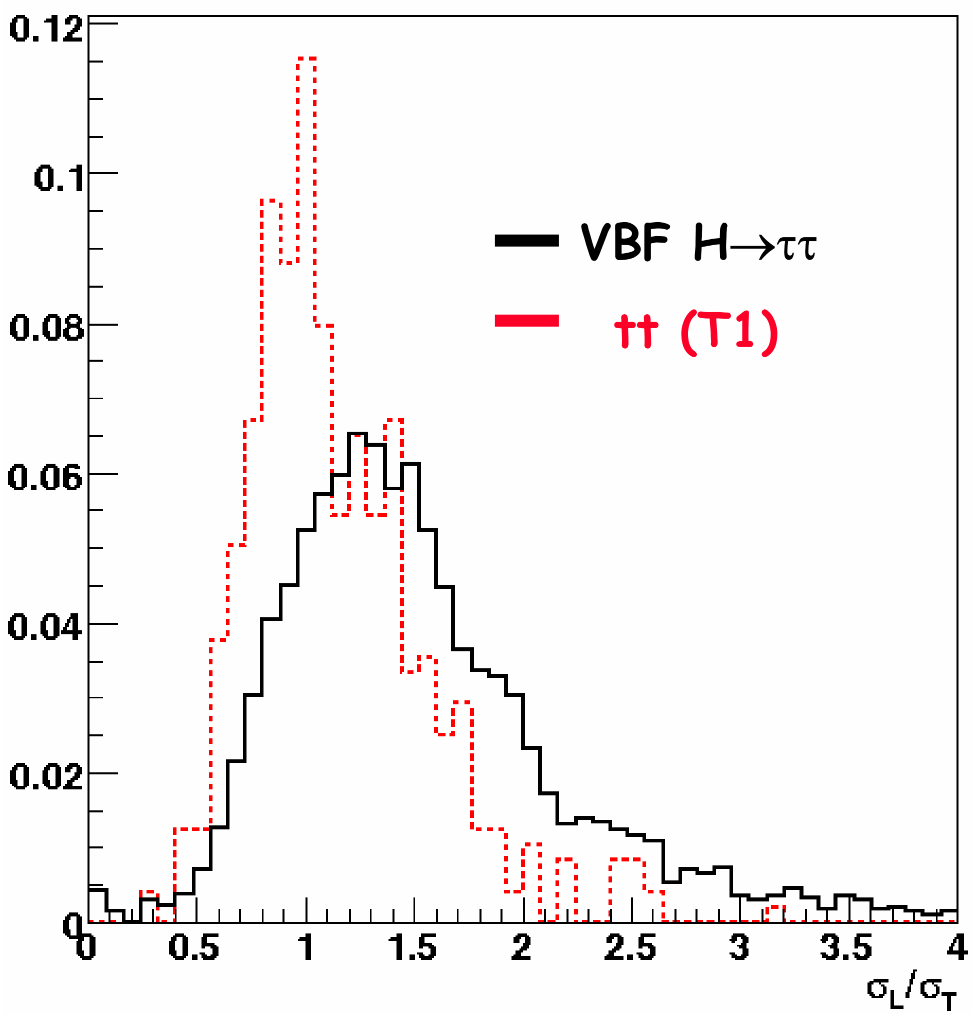


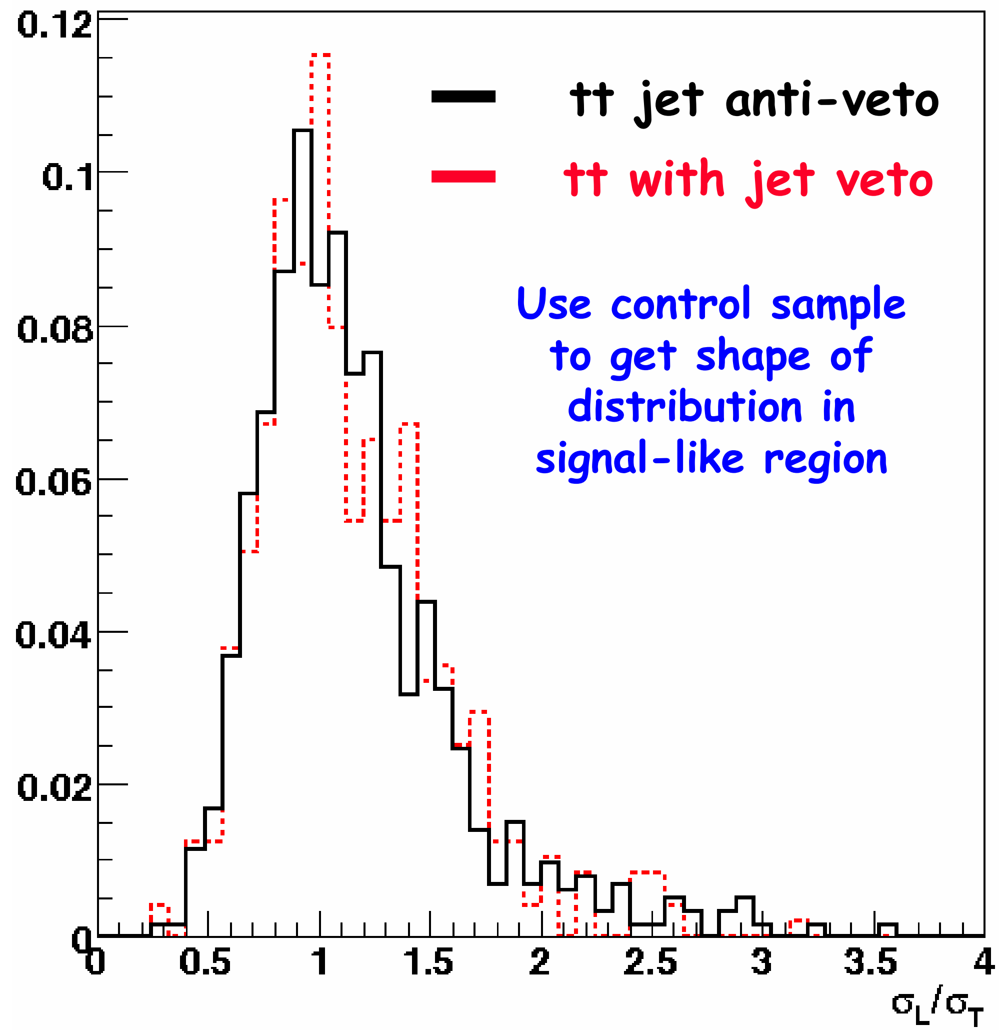
σ_L/σ_T For High P_T di-jets and SUSY (SU1)



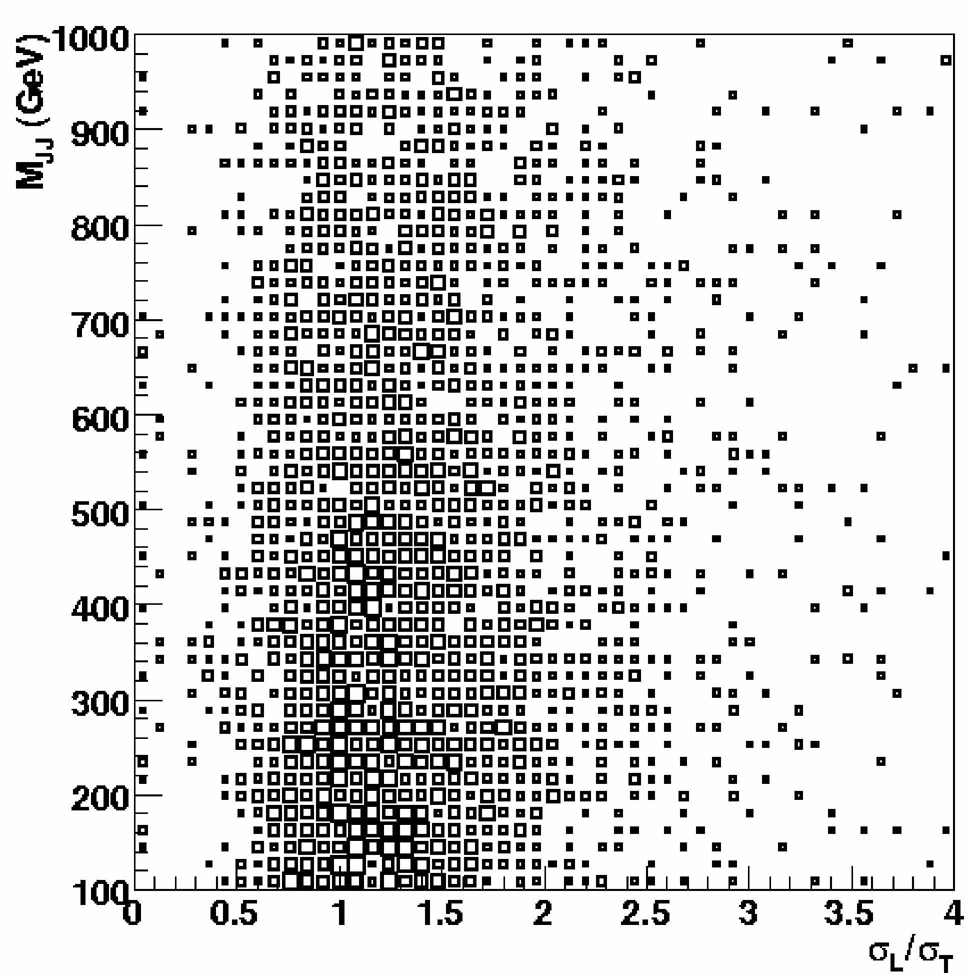


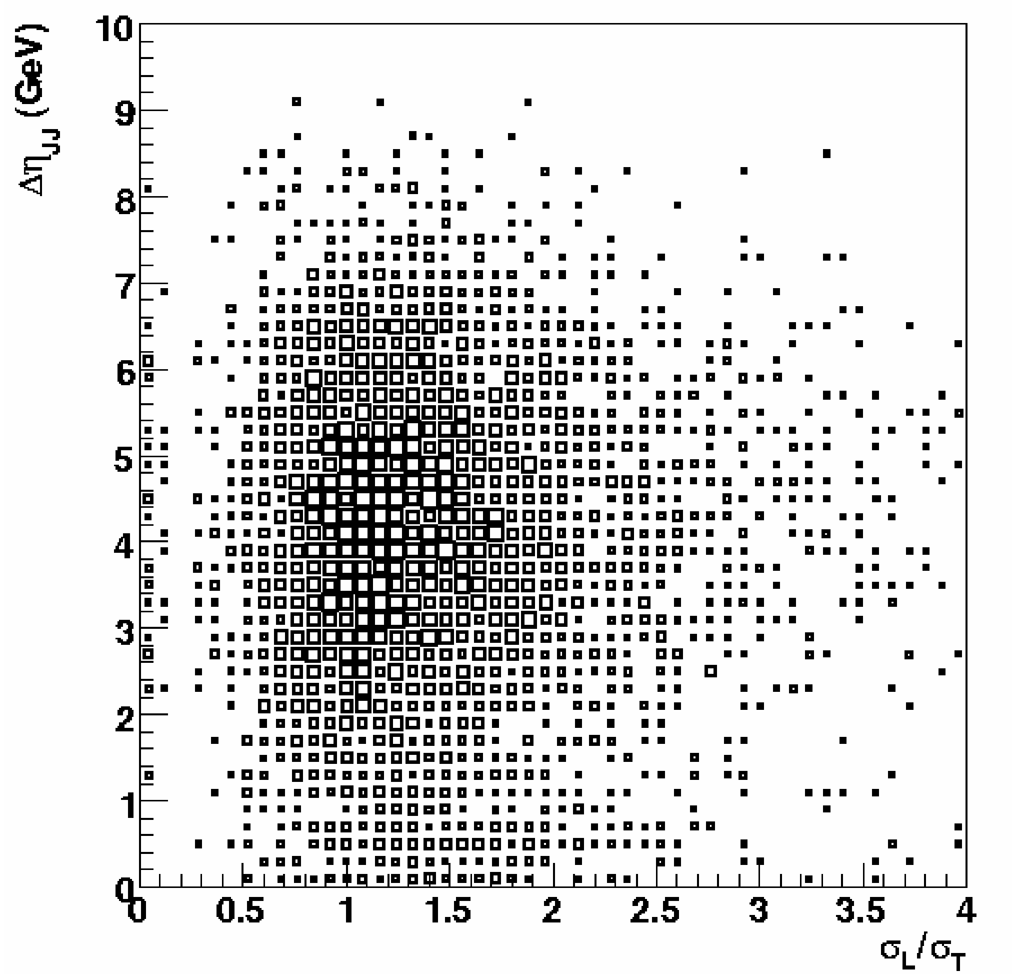
σ_L/σ_T with MET>30 GeV

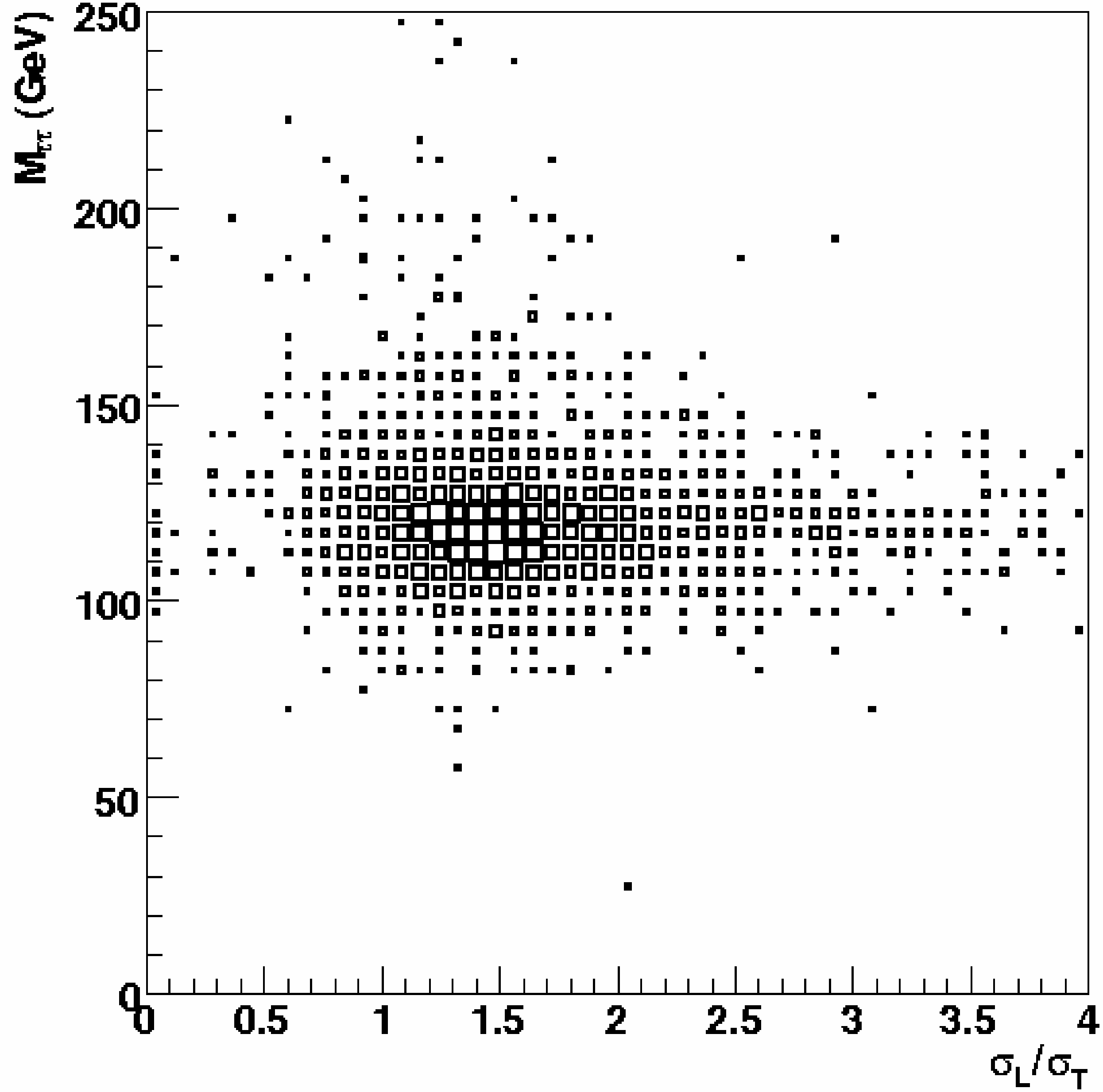




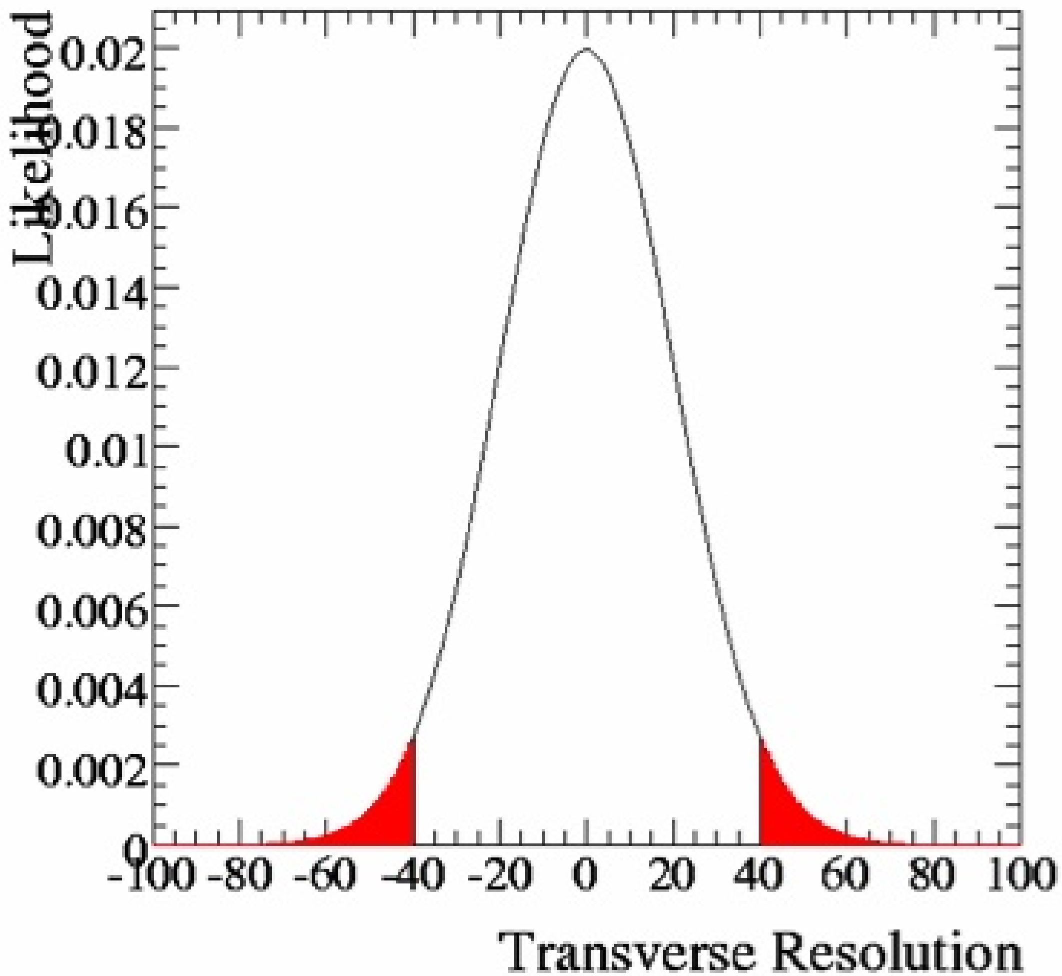
Do not observe obvious correlation between σ_L/σ_T and the leading discriminating variables






250
200
150
100
50
0
$M_{\tau\tau}$ (GeV)
0
0.5
1
1.5
2
2.5
3
3.5
4
σ_L/σ_T

The p-value Counterpart in the Transverse Direction



- In order to make use of the transverse information, in analogy to the p-value defined in the longitudinal direction, we can define a double-sided p-value (the D parameter) region to compute the transverse significance
- Have to determine where to set the boundary of the critical regions — MET scaled by some factor maybe a candidate
- Study is still going on

Outlook and Plans

- The concept of Missing E_T significance is widely used in analyses in pp collisions
  - It adds discrimination and robustness to Missing E_T. A combination of both is more powerful
    - Necessary to understand the resolution of objects
- A package (METSig) is available in ATHENA
  - A more sensitive definition of Missing ET significance has been developed. It seems to be superior to the D0 discriminator
  - Add information in the transverse direction to MET
- METSig is a new tool and it needs to be understood
  - Need collaborators to develop this effort