Introduction to the SM (6)

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Last lecture

- Define the SM
- Discuss the SSB sector
- Leptons and gauge bosons spectrum and interactions

Today: The quark sector of the SM

HW: Some explicit calculations

- Using $\phi(1,2)_{1/2}$ write $D^\mu\phi$

$$D^\mu\phi = \left(\partial^\mu + \frac{i}{2} g W_a^\mu \sigma_a + \frac{i}{2} g' B^\mu\right)\phi$$

- Calculate the mass matrix of the gauge bosons

$$|D^\mu\phi|^2 \ni \frac{1}{8}\left|\begin{pmatrix} gW_3 + g'B & g(W_1 - iW_2) \\ g(W_1 + iW_2) & -gW_3 + g'B \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix}\right|^2$$

$$= \frac{g^2v^2}{8}\left[|W_1 - iW_2|^2 + |W_3 - (g'/g)B|^2\right]$$

$$= \frac{g^2v^2}{4} W^+W^- + \frac{(g^2 + g'^2)v^2}{8}(\cos\theta_W W_3 - \sin\theta_W B)^2$$

Experimental tests of $\rho = 1$

- From the $\rho = 1$ relation

$$\frac{m_W^2}{m_Z^2} = \cos^2\theta_W \approx \left(\frac{80.4}{91.2}\right)^2 \approx 0.77 \quad \Rightarrow \quad \sin^2\theta_W \approx 0.23$$

- Z decays to leptons

$$\Gamma(Z \to \ell\ell) \;\sim \sum_{L,R} (T_3 - Q\sin^2\theta_W)^2$$

$$\Gamma(Z \to \ell^+\ell^-) \;\sim (1/2 - \sin^2\theta_W)^2 + (\sin^2\theta_W)^2 \sim 1/8$$

$$\Gamma(Z \to \nu\bar{\nu}) \;\sim (1/2)^2 \sim 1/4 \quad \Rightarrow \quad r \equiv \Gamma_\ell/\Gamma_{\rm inv} \sim 1/6$$

- PDG: $\Gamma_\ell = 3.37\%$ and $\Gamma_{\rm inv} = 20.00\% \quad \Rightarrow \quad r \sim 1/6$

Quarks

Basic of basis rotation

- We want to move to another basis
- Usually, we like to rotate fields with the same QN (same representation). Why?

$$\psi' = U\psi \qquad UU^\dagger = 1$$

- What it does to the kinetic term, $\bar\psi^i \delta_{ij} D_\mu \gamma_\mu \psi^j$?
- What it does to the mass term, $\bar\psi^i_L m_{ij} \psi^j_R$?

$$\bar\psi^i_L m_{ij} \psi^j_R = \bar\psi^i_L U U^\dagger m_{ij} V V^\dagger \psi^i_R = \bar\psi'^i_L m'_{ij} \psi'^j_R$$

- How we used it for the leptons?

The CKM matrix

$\mathcal{L}_{Yuk} \sim \phi QU + \phi QD$. After SSB we define $Q = (U \;\; D)$

$$\mathcal{L} \sim g\,\bar{u}_L^i \delta_{ij} d_L^j\, W + m_{ij}^D \bar{d}_R^i d_L^j + m_{ij}^U \bar{u}_R^i u_L^j$$

- The mass matrices, m_{ij}^F, are general complex matrices
- We can diagonalize them and move to the mass basis

$$m_{diag} = V_L m V_R^\dagger \qquad VV^\dagger = 1 \qquad V_L \neq V_R$$

$$(d_L)_i \to (V_L^D)_{ij} (d_L)_j \qquad (d_R)_i \to (V_R^D)_{ij} (d_R)_j$$

- The kinetic term, photon and Z couplings are invariant

$$\mathcal{L}_\gamma \sim \bar{d}_i \delta_{ij} d_j \;\to\; \bar{d}_i V \delta_{ij} V^\dagger d_j \sim \bar{d}_i \delta_{ij} d_i$$

CKM

- For the W the rotation to the mass basis is important

$$\mathcal{L}_W \sim \bar{u}_L^i \delta_{ij} d_L^j \;\to\; \bar{u}_i V_L^U \delta_{ij} V_L^{D\dagger} d_j \sim \bar{u}_i V d_i$$

- V is the CKM matrix

$$V = V_L^U V_L^{D\dagger}$$

- The point is that we cannot have m_U, m_D and the couplings to the W diagonal at the same basis
- In the mass basis the W interaction change flavor, that is, flavor and generation number is not conserved
- We can work in another basis. The point is that at most 2 out of the 3 matrices can be diagonal

Implication of the CKM matrix

$$\mathcal{L} \ni V_{ij}\bar{u}_i\gamma_\mu d_j W^\mu$$

- Unlike for the leptons the flavor $U(1)$s are broken
- The CKM is parametriezed by 3 angles and one phase
- Numerically, $V \sim 1$ ($V_{us}/V_{ud} \approx 0.22$)
- We can probe the values of the entries in many ways

$$\frac{\Gamma(B \to X_c \ell\bar{\nu})}{\Gamma(B \to X_u \ell\bar{\nu})}$$

- HW: Using the PDG, estimate $|V_{cb}/V_{ub}|$. Any ideas how to make it more precise?

FCNCs

FCNC

FCNC=Flavor Changing Neutral Current

- Diagonal couplings vs universal couplings
- $\mu \to 3e$ vs $\mu \to e\nu\bar{\nu}$. Also $b \to c\ell\nu$ vs $b \to s\ell^+\ell^-$
- In Nature, FCNC are highly suppressed (HW:check!)
  - Historically, $K \to \mu\nu$ vs $K_L \to \mu\mu$
  - FCNSs in charm, B and top are also small
- In the SM there are no FCNCs at tree level. Very nice!
- The coupling of the four neutral bosons of the SM: g, γ, Z, h, are diagonal
- We can have FCNC at one loop (two charged current interactions give a neutral one)

Photon and gluon tree level FCNC

- For exact gauge interactions the couplings are always diagonal. It is part of the kinetic term

$$\partial_\mu \delta_{ij} \to (\partial_\mu + iqA_\mu)\delta_{ij}$$

- Symmetries are nice...
- In any extension of the SM the photon and gluon couplings are flavor diagonal

Higgs tree level FCNC

- The Higgs is a possible source of FCNC
- With one Higgs doublet, the mass matrix is aligned with the Yukawa. $\phi \sim v + H$

$$\mathcal{L}_m \sim Y\, v\, \bar{d}_L d_R \qquad \mathcal{L}_{int} \sim Y H \bar{d}_L d_R$$

- With two doublets we have tree level FCNC

$$\mathcal{L}_m \sim \bar{d}_L (Y_1 v_1 + Y_2 v_2) d_R \qquad \mathcal{L}_{int} \sim H_1 \bar{d}_L Y_1 d_R$$

- There are “ways” to avoid it, by imposing extra symmetries

Z exchange FCNC

- For broken gauge symmetry there is no FCNC when: "All the fields with the same QN in the unbroken symmetry also have the same QN in the broken part"
- In the SM the Z coupling is diagonal since all $q = -1/3$ RH quarks are $(3,1)_{-1/3}$ under $SU(2) \times U(1)$
- What we have in the couplings is

$$\bar{d}_i \, (T_3)_{ij} \, d_j \;\to\; \bar{d} \, V \, (T_3)_{ij} \, V^\dagger d_j \qquad V T_3 V^\dagger \propto I \;\text{ if }\; T_3 \propto I$$

- Adding quarks with different representations can generate tree level FCNC Z couplings, like $\psi_L(3,1)_{-1/3}$
- Same condition for new neutral gauge bosons (usually denoted by Z')

CPV

C, P and CP

Discrete symmetries of space-time

- CPT is a good symmetry of any theory we discuss
- How to look for P violation?
- In the SM C and P are broken by construction
- CP, however, may or may not be broken. It is broken if δ_{KM} is not zero

CP violation

A simple “hand wave” argument of why CP violation is given by a phase

- It is all in the $+h.c.$ term

$$Y_{ij}\left(\bar{Q}_L\right)_i \phi\left(D_R\right)_j + Y_{ji}^*\left(\bar{D}_R\right)_j \phi^\dagger\left(Q_L\right)_i$$

- Under CP

$$Y_{ij}\left(\bar{D}_R\right)_j \phi^\dagger\left(Q_L\right)_i + Y_{ji}^*\left(\bar{Q}_L\right)_j \phi\left(D_R\right)_i$$

- CP is conserved if $Y_{ij} = Y_{ij}^*$
- Not a full proof, since there is still a basis choice...

What is CP?

- A symmetry between a particle and its anti-particle
- CP is violated if we have

$$\Gamma(A \to B) \neq \Gamma(\bar{A} \to \bar{B})$$

- In the SM it is closely related to flavor
- It is a very small effect in Nature
- It is not easy to detect CPV
  - Need interference of two (or more) amplitudes
  - Need 2 amplitudes with different weak and strong phases
  - CPT $\Rightarrow \Gamma_X = \Gamma_{\bar{X}}$. Thus, we need at least two modes with CPV

All these phases

- Weak phase (CP-odd phase)
  - Phase in $\mathcal{L}$
  - In the SM they are only in the weak part so they are called weak phases

$$CP(Ae^{i\phi}) = Ae^{-i\phi}$$

- We like to use CPV observables to measure and test the phase of the CKM

Strong phase

- Strong phase (CP-even phase). Do not change under CP

$$CP(Ae^{i\delta}) = Ae^{i\delta}$$

  - Due to time evolution

$$\psi(t) = e^{-iHt}\psi(0)$$

  - They are also due to intermediate real states, and have to do with "rescattering" of hadrons
  - Such strong phases are very hard to calculate

Why we need the two phases?

Intuitive argument

- If we have only one amplitude $|A|^2 = |\bar{A}|^2$
- Two amplitudes but with a different of only of weak phase
$$\left|A + be^{i\phi}\right|^2 = \left|A + be^{-i\phi}\right|^2$$
- When both phases are not zero, the rates are not the same (do it for HW!)

What to do with CPV

- The basic idea is to find processes where we can measure CPV
- In some cases they are very clean so we get sensitivity to the CKM matrix elements and phase
- Examples: $K_L \to \pi\pi$ and $B(t) \to \psi K_S$
- We can use flavor physics observables to test the CKM picture of the SM