August 28, 2014

Cornell University, Department of Physics

PHYS 7646, Elementary particle II, HW # 1, due: 9/4/2014

General remark: This homework is more a review of topics that we just touched upon.

Question 1: Exotic light quarks

We consider a model with the gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ spontaneously broken by a single Higgs doublet into $SU(3)_C \times U(1)_{EM}$. The quark sector, however, differs from the standard model one as it consists of three quark flavors, that is, we do not have the c, b and t quarks. The quark representations are non-standard. Of the left handed quarks, $Q_L = (u_L, d_L)$ form a doublet of $SU(2)_L$ while s_L is a singlet. All the right handed quarks are singlets. All color representations and electric charges are the same as in the standard model.

1. Write down (a) the gauge interactions of the quarks with the charged W bosons (before SSB); (b) the Yukawa interactions (before SSB); (c) the bare mass terms (before SSB); (d) the mass terms after SSB.

2. Show that there are four physical flavor parameters in this model. How many are real and how many imaginary? Is there CP violation in this model? Separate the parameters into masses, mixing angles and phases.

Question 2: Question 2.1 of the notes

Question 3: Question 2.2 of the notes

Question 4: Lepton universality

Here we consider muon and tau decays in the SM.

1. Find in the PDG the main decay mode of the muon. What is its width?

2. Find in the PDG the bound on the decay

$$\Gamma(\mu \to e\gamma). \tag{1}$$

   What is the SM prediction to this mode? Give a short explanation not just a number.

3. Draw the Feynman diagram for the leading muon decay.

4. We now move to tau decays. Based on lepton universality, what do you expect for the following ratios

$$\frac{\Gamma(\tau \to e\nu\bar{\nu})}{\Gamma(\tau \to \mu\nu\bar{\nu})}, \qquad \frac{\Gamma(\mu \to e\nu\bar{\nu})}{\Gamma(\tau \to e\nu\bar{\nu})} \tag{2}$$

Compare your results with the PDG and explain any small deviation that you find compare to your predictions.

Question 5: Pion decay

Consider the decays

$$\pi^+ \to \ell^+ \nu_\ell. \tag{3}$$

In class we mentioned that for these decays

$$R \equiv \frac{\Gamma(\pi^+ \to e^+\nu_e)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2 . \tag{4}$$

The calculation can be found in many books. Here we followed Greiner and Müller (2nd edition) pages 205–210. The pion decay amplitude is given by

$$\mathcal{M} = \sqrt{2} G_F f_\pi q_\alpha \bar{u}_\ell(p, s)\gamma^\alpha(1 - \gamma_5)v_{\nu_\ell}(k, t). \tag{5}$$

where $\ell = e, \mu$ denotes the lepton type, G_F and f_π are constants independent of the lepton flavor, q, p, k are the momenta of the pion, charged lepton, and neutrino, respectively and s and t are the spin index of the relevant spinors.

1. Repeat the calculation of R, assuming that the neutrinos are massive. Namely, consider a case where ν_e has mass m_1 and ν_μ has mass m_2. Use the following formula for a decay rate of a particle of mass M that decays into two particles of masses m_1 and m_2

$$\Gamma = \frac{1}{16\pi} \sum |\mathcal{M}|^2 \frac{|\mathrm{p_i}|}{M^2} = \frac{\sqrt{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}}{32\pi M^3} \sum |\mathcal{M}|^2.$$

   Check that your result agrees with (4) in the case of massless neutrinos.

2. Using experimental data derive a bound on the electron neutrino mass. For this assume that $m_2 = 0$ and expand the branching ratio up to $\mathcal{O}(m_1^2)$ (You can use *Mathematica*, *Matlab*, etc. for that). Further assume that the experimental error is 4×10^{-3} and derive the bound. (Note: This bound is not very strong compare to other bounds, so this exercise is academic only.)