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Cornell University, Department of Physics

PHYS 7646, Elementary particle II, HW # 4, due: 10/28/14

Question 1: Exotic light quarks, again

We consider the model we did in HW1. The gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ spontaneously broken by a single Higgs doublet into $SU(3)_C \times U(1)_{EM}$. The quark representations are such that $Q_L = (u_L, d_L)$ form a doublet of $SU(2)_L$ while s_L is a singlet. All the right handed quarks are singlets. All color representations and electric charges are the same as in the standard model.

1. Are there photon and gluons FCNC's? Support your answer by an argument based on symmetries.
2. Write down the gauge interactions of the quarks with the Z boson in both the interaction basis and the mass basis. (You do not have to rewrite terms that do not change when you rotate to the mass basis. Write only the terms that are modified by the rotation to the mass basis.) Are there generally tree level Z exchange FCNC's?
3. We assume that the masses of the particles and the value of the Cabibbo angle are as found in nature and that the leptons are described by the SM. Then, in this model we can have process like $K_L \to \mu^+\mu^-$. Estimate its rate (normalized it to $K^+ \to \mu^+\nu$).
4. Explain why this result practically ruled out the model.

Question 2: Two Higgs doublet model

Consider the two Higgs doublet model (2HDM) extension of the SM. In this model we add a Higgs doublet to the SM fields. Namely, instead of the one Higgs field of the SM we now have two, denoted by ϕ_1 and ϕ_2. For simplicity you can work with two generations when the third generation is not explicitly needed.

1. Write down (in a matrix notation) the most general Yukawa potential of the quarks.
2. Carry out the diagonalization procedure for such a model. Show that the Z couplings are still flavor diagonal.
3. In general, however, there are FCNCs in this model mediated by the Higgs bosons. To show that, write the Higgs fields as $\Re(\phi_i) = v_i + h_i$ where $i = 1, 2$ and $v_i \neq 0$ is the

vev of ϕ_i, and define $\tan\beta = v_2/v_1$. Then, write down the Higgs–fermion interaction terms in the mass basis. Assuming that there is no mixing between the Higgs fields, you should find a non diagonal Higgs fermion interaction terms.

4. Since there are FCNCs in this model processes like $b \to s\ell^+\ell^-$ can proceed at tree level. Assume that $\tan\beta \sim 1$ and $m_{H_i} \sim m_W$ and give a very rough estimate of the ratio
$$\frac{\Gamma(b \to s\mu^+\mu^-)}{\Gamma(b \to c\mu^-\bar{\nu})} \tag{1}$$
(For the numerical values use $m_b = 4.3$ GeV and $V_{cb} = 0.04$.)

5. The current upper bound on this ratio is 5×10^{-6}. Can we already probe this model using this ratio?

6. Can you find a symmetry that will forbid the Higgs exchange FCNCs? In particular, try to find a symmetry that will couple ϕ_1 only to the up type quarks, and ϕ_2 to the down type quarks.

Question 3: Kaons

Here we study some properties of the kaon system.

1. Explain why $y_K \approx 1$.

2. In a hypothetical world where we could change the mass of the kaon without changing any other masses, how would the value of y_K change if we made m_K smaller or larger.

Question 4: Mixing beyond the SM

Consider a model without a top quark, in which the first two generations are as in the SM, while the left–handed bottom (b_L) and the right–handed bottom (b_R) are $SU(2)$ singlets.

1. Draw a tree-level diagram that contributes to $B - \bar{B}$ mixing in this model.

2. Is there a tree-level diagram that contributes to $K - \bar{K}$ mixing?

3. Is there a tree-level diagram that contributes to $D - \bar{D}$ mixing?

Question 5: Mixing formalism

In this question, you are asked to develop the general formalism of meson mixing.

1. Show that the mass and width differences are given by

$$4(\Delta m)^2 - (\Delta\Gamma)^2 = 4(4|M_{12}|^2 - |\Gamma_{12}|^2), \qquad \Delta m \Delta\Gamma = 4Re(M_{12}\Gamma_{12}^*), \tag{2}$$

and that

$$\left|\frac{q}{p}\right| = \left|\frac{\Delta m - i\Delta\Gamma/2}{2M_{12} - i\Gamma_{12}}\right|. \tag{3}$$

2. When CP is a good symmetry all mass eigenstates must also be CP eigenstates. Show that CP invariance requires

$$\left|\frac{q}{p}\right| = 1. \tag{4}$$

3. In the limit $\Gamma_{12} \ll M_{12}$ show that

$$\Delta m = 2|M_{12}|, \qquad \Delta\Gamma = 2|\Gamma_{12}|\cos\theta, \qquad \left|\frac{q}{p}\right| = 1. \tag{5}$$

4. Show that

$$\begin{aligned}\Gamma(B \to f)[t] = |A_f|^2 e^{-\tau} \Big\{ &(\cosh y\tau + \cos x\tau) + |\lambda_f|^2 (\cosh y\tau - \cos x\tau) \\ &-2Re\left[\lambda_f(\sinh y\tau + i \sin x\tau)\right] \Big\}, \\ \Gamma(\bar{B} \to f)[t] = |\bar{A}_f|^2 e^{-\tau} \Big\{ &(\cosh y\tau + \cos x\tau) + |\lambda_f|^{-2} (\cosh y\tau - \cos x\tau) \\ &-2Re\left[\lambda_f^{-1}(\sinh y\tau + i \sin x\tau)\right] \Big\},\end{aligned} \tag{6}$$

5. Conisder the case where $|\lambda| = 1$ and f is a CP eigenstate and show that

$$\mathcal{A}_f(t) = Im\lambda_f \sin(x\tau) = \sin\left[\arg(\lambda_f)\right]\sin(\Delta m t). \tag{7}$$

6. Show that when $\Delta\Gamma = 0$ and $|q/p| = 1$

$$\begin{aligned}\Gamma(B \to X\ell^-\bar{\nu})[t] &= e^{-\Gamma t}\sin^2(\Delta m t/2), \\ \Gamma(B \to X\ell^+\nu)[t] &= e^{-\Gamma t}\cos^2(\Delta m t/2).\end{aligned} \tag{8}$$