Question 1: Neutrino oscillations

Here you are asked to derive some of the basic formulas of neutrino oscillations.

1. Consider Eq. (3.7) of my TASI 2002 lectures (a link is on the web site). In (3.7), I assumed CP conservation. You are asked not to assume it, and then write the same formula for anti-neutrino oscillations. As a check, take the CP limit and see that you recover (3.7). Note however that in (3.7) there is a typo and a factor of 2 is missing. Make sure you correct this typo.

2. Derive an expression for the time reversal process different

\[ \Delta_T \equiv P(\nu_e \rightarrow \nu_\mu) - P(\nu_\mu \rightarrow \nu_e) \] (1)

and for the CP reversal one

\[ \Delta_{CP} \equiv P(\nu_e \rightarrow \nu_\mu) - P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu) \] (2)

and show that in the CP limit, which you can take as the case when \( U \) is real, \( \Delta_T = \Delta_{CP} = 0 \).

3. Due to CPT we know that \( \Delta_T - \Delta_{CP} = 0 \). Check that this is indeed the case.

4. Consider a case where the production state is \( \nu_e \) and the detection is done via elastic scattering. In that case the detected neutrino is some superposition of flavor eigenstates which we define as \( \nu_d \). Find \( P(\nu_e \rightarrow \nu_d) \) as a function of distance. For simplicity you can assume only two generations.

Question 2: Matter effects

1. Derive Eq. (3.16) from the notes.

2. Show that to linear order in \( x \) matter effects cancel in the oscillation probability.
Question 3: Solar neutrinos

One possible solution for the solar neutrino problem is the so-called “vacuum oscillation.” The idea is that the mass difference is very small, and then matter effect can be neglected, and all we care about is the oscillation of the neutrinos while traveling to Earth.

1. Estimate the needed $\Delta m^2$ for such a solution to work.

2. One prediction of such an idea is seasonal variation. Explain how this can be observed and estimate the magnitude of the effect.