Cornell University, Department of Physics

PHYS 7646, Elementary particle II, HW # 7

Question 1: Coupling unification

1. Using one loop RGE running and the values of $\alpha_i(M_z)$ $(i = 1, 2, 3)$, calculate the unification scale, M_{GUT} and the unifued value of the coupling, $\alpha_5(M_{\mathrm{GUT}})$. You should give three different results for unification of any two combination of couplings.

2. You can improve the unification of couplings by adding particles to the SM. We now assume that we add n Higgs doubles. Discuss how it changes the picture of unification.

Question 2: SU(5) GUTs and doublet-triplet splitting

Assume that the SU(5) GUT symmetry is broken by the VEV of an adjoint scalar Σ with VEV

$$\langle \Sigma \rangle = v_1 \begin{pmatrix} 3 & & & & \\ & 3 & & & \\ & & -2 & & \\ & & & -2 & \\ & & & & -2 \end{pmatrix}. \tag{1}$$

1. Calculate the mass of the X and Y gauge bosons.

2. Integrate out the X and Y gauge bosons to determine the effective 4-fermi interactions of the SM fermions relevant for baryon number violating processes at low energies.

3. Assume that there is a coupling between the SM Higgs H (in the 5 of SU(5)) and the Σ of the form
$$V(\Sigma, H) = \lambda_1 H^\dagger \Sigma H + \mu^2 H^\dagger H. \tag{2}$$

   Derive a relation that has to hold between λ_1, v_1 and μ^2 to ensure that the SM doublet Higgs does not pick up a mass of order $v_1 \sim M_{GUT}$. Is this relation justifiable in any way? Assuming that this relation holds, what will be the mass of the color triplet in H?

Question 3: Flipped SU(5)

Consider an alternative assignment of fermions to the 5-dimensional representation of SU(5). We add a singlet neutrino and the 16 DOFs are assigned in the following way

$$5 \in (L, U^c), \qquad 10 \in (N, Q, D^c), \qquad 1 \in E^c \tag{3}$$

1. Explain why the extra $U(1)_X$ is needed.

2. Obtain the X-quantum numbers of the different multiplets, and show how hypercharge is express as function of $U(1)_X$ and T_{12}.

3. Discuss how fermion masses arise from spontaneous symmetry breaking in this model.